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A STUDY OF THE CAPACITOR MOTOR

T. J. JAKUBOWSKI

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A STUDY OF THE CAPACITOR MOTOR

A compilation of basic material
on the capacitor motor

by

Thaddeus John Jakubowski
Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE
IN ELECTRICAL ENGINEERING

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the thesis requirements for the degree of
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from the
United States Naval Postgraduate School

Chairman
Department of Electrical Engineering

Approved:

Academic Dean

18039

PREFACE

To the author's knowledge, there has been no extensive compilation of basic information on the capacitor motor; therefore, the incentive for this paper has been to compile as much of this material as time will permit.

The author hopes that this paper is sufficiently inclusive to serve as a broad reference on capacitor motors.

Monterey, California
June 1952

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CHAPTER I

INTRODUCTION

Some fifty years ago Doctor Steinmetz and his associates developed a capacitor motor and secured a number of patents on it. Some of these motors were actually built and placed in service, but their manufacture was soon abandoned largely because of the difficulty, at that time, of securing capacitors of sufficiently reliable construction.

After this experience, there was a long period in which little interest in them was shown except for an occasional review which always brought the same answer; that is until a cheap and reliable capacitor could be found, the capacitor motor would be commercially impractical.

In the late twenties great strides had been made in the manufacture of capacitors. The experience gained in their manufacture for the radio industry, as well as for power-factor correction, had been largely responsible for the improvements made. On the basis of this experience, there had been a definite effort made to develop a capacitor which would be satisfactory for use with motors, and the result was that at that time, the cheap and reliable capacitators for which the motor manufacturers

had been waiting were available.

Thus at that time, with the rapid increase which had taken place in the number of motors applied to such semi-continuous operated domestic loads as household refrigerators and oil burners, there had risen a demand for fractional horsepower motors of very high quality, such motors operating on house lighting circuits as they do and running at all times of night and day, had to be quiet, have high operating characteristics, low starting current to prevent flicker of lights, and no radio interference. Of all types of single phase-fractional horsepower motors available, the capacitor motor seemed best suited for this sort of service. It not only has all of the characteristics necessary, but in addition is probably the simplest and most reliable of all high quality single-phase motors.

CAPACITATOR MOTOR DEFINED. The following definition of capacitor motor, was published by Nema, May 1, 1946;

"A capacitor motor is a single-phase induction motor with a main winding arranged for direct connection to a source of power and an auxiliary winding connected in series with a capacitor."

"There are three types of capacitor motors:

1.) Capacitor-start motor.

A. Capacitor-start motor is a capacitor motor

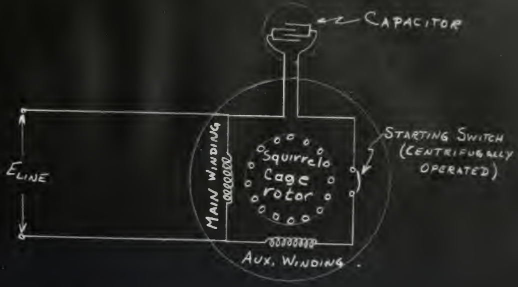


Fig. 1A - The capacitor-start motor

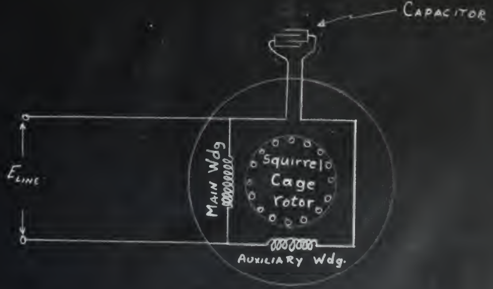


Fig. 1B - The permanent-split capacitor motor

in which the capacitor phase is in the circuit only during the starting period.

2.) Permanent-split Capacitor motor.

A permanent-split capacitor motor is a capacitor motor having the same value of capacitance for both starting and running conditions.

3.) Two-Value Capacitor Motor.

A two-value capacitor motor is a capacitor motor using different values of effective capacitance for the starting and running connections."

The two-value capacitor motor gained great commercial importance in the early 1930's for applications requiring high locked-rotor torque, such as refrigerators, compressors, and stokers.

Essential Parts of the Two-value Motor. There are two major types of two-value capacitor motors: one uses a capacitor-transformer unit, and the other uses two capacitors. The two types will be discussed in order.

(a) Motor Using a Capacitor-Transformer Unit.

A two value motor using a capacitor-transformer unit is represented diagrammatically in Figure 1D. To all intents and purposes, the windings are the same as for the capacitor-start motor, and the rotor is of squirrel-cage construction. A

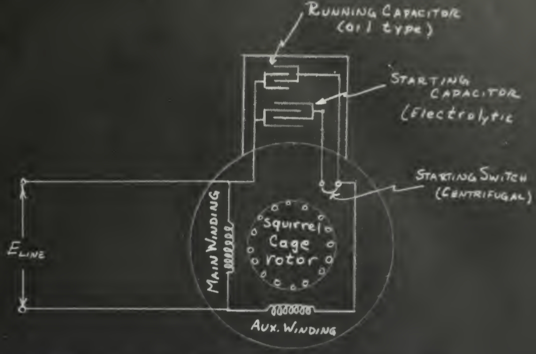


Fig. 1C - The two-value capacitor motor using two capacitors.

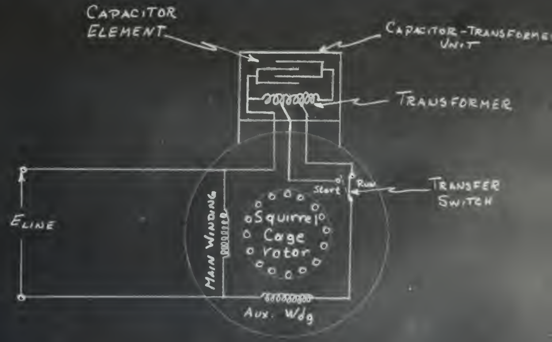


Fig. 1D - The two-value capacitor motor using a capacitor-transformer unit.

difference in the starting switch will be noted, however, for this motor uses a "transfer switch" which is equivalent to a single-pole double-throw switch. (The split phase and capacitor-start motors use a switch which is equivalent only to a single-pole single-throw switch.) By means of this transfer switch, a high voltage (approximately 600 to 800 volts) is impressed for the starting purposes across the capacitor element: this voltage is then reduced to approximately 300 volts for continuous running. This change in voltage is effected by changing taps on the primary side of the auto transformer and corresponds to changing the effective microfarads in series with the auxiliary winding.

(b) Motor Using Two Capacitors.

A two value capacitor motor using two separate capacitors is illustrated diagrammatically in Figure 1C. The arrangement of windings and switch in this motor is identical with the arrangement used in the capacitor-start motor illustrated in Figure 1A. The difference between these two motors is that the two-value motor has a running capacitor permanently connected in series with the auxiliary; the starting capacitor is paralleled with the running capacitor only during the

starting period. The running capacitor is usually of the paper-spaced oil filled type, rated at 330 volts alternating current, continuous operation. Castor oil, mineral oil, Dykanol, Inerteen, Insulatum, and Pyranol are variously used as impregnating medium. The capacities range from 3 to 16 mfd. The electrolytic capacitors for 115 volt 60-cycle motors range in size from 5 mfd or less to 300 mfd or more.

Effect of the Running Capacitor

It was pointed out in the preceding paragraph that the motor of Figure 1C is merely the capacitor start motor of Figure 1A with a running capacitor permanently connected in the circuit. Considering any given motor, the effect of adding this capacitor is to:

1. Increase the breakdown torque from 5 to 30 percent.
2. Improve the full-load efficiency and power factor.
3. Reduce the noise under full load operating conditions.
4. Increase the locked rotor torque 5 to 20 percent.

The running capacitor improves performance because the rotating field of a single-phase induction motor may be considered as comprised of two components pulsating fields, a main field and a cross field spaced 90 electrical degrees apart and differing in time phase by 90 degrees; the main-field component is set up directly by the main winding and

the cross field by the rotor. When the auxiliary winding is connected across the line in series with a running capacitor of the proper value, the current drawn by the auxiliary winding leads the current drawn by the main winding by approximately 90 degrees. The net result is that the auxiliary winding sets up a part or all of the cross field, thereby reducing or eliminating the magnetizing currents in the rotor and the accompanying copper losses.

		AS 1-Ø MOTOR		AS CAP MOTOR	
32	r_2/X_0 hot $= 1.15 r_2/X_0$	r_2/X_0	0.0505	r_2/X_0	0.0505
33	Slip @ Full Load	s	0.0417	s	0.0333
34	Slip of backward field	2-s	1.958	2-s	1.967
35	sR_f		0.711		0.531
36	$(2-s)\ R_b$		1.753		1.753
37	Amperes @ Full Load	I_1	3.75		
38	Sec. $I^2\ R(f)$	$I_1^2 \cdot (35)$	10.0	$(30) \cdot (35)$	7.25
39	Sec. $I^2\ R(b)$	$I_1^2 \cdot (36)$	24.6	$(31) \cdot (36)$	1.51
40	Main wdg cu loss	$I_1^2\ r_{1m}$	23.6	$I_{1m}^2\ r_{1m}$	9.0
41	Aux wdg cu loss			$I_{1a}^2\ r_{1a}$	4.7
42	Cap unit loss			$I_{1a}^2\ R_c$	10.2
43	Core loss + friction	(23)	31.1	(23)	31.1
44	Total losses @ F.L.		89.3		63.3
45	Output		186.4		186.4
46	Output plus losses		275.7		249.7
47	Efficiency by losses		67.6		74.7

HP 1/4 Volts 110 Cycles 60 Poles 4

The effect of the addition of a running capacitor is illustrated in the preceding tabulation of a section of a test analysis form.

In short, the effect of the running capacitor is to make the motor perform more nearly like a two-phase motor, particularly at one value load. It is not possible to duplicate two-phase motor performance at all load values with a single value of capacitance, however, because a different value of capacitance would be required for each different load.

In regards to the future of the capacitor motor, Mr. Bewley[2] in 1949 made the following assertion: "The capacitor motor is a split-phase machine in which the auxiliary winding remains permanently in circuit, and the motor is intended to have the running characteristics of a two phase motor. Because it can be designed to have high starting torque, as well as good running characteristics (including leading power-factor), and because its speed can be controlled rather easily, this motor seems destined to supplant some of the other types of single phase motors."

CHAPTER II

THE REVOLVING FIELD THEORY (7)

In the revolving field theory of single-phase motors the pulsating sinusoidal flux produced by the stator winding is resolved into two equal sinusoidal waves of flux gliding in opposite directions around the periphery of the air gap at synchronous speed. The effect upon the stator produced by each of these revolving fluxes and the induced forward and backward revolving rotor fluxes can be represented by a parallel circuit of two branches. One branch is the magnetizing reactance and the other is composed of the secondary leakage reactance in series with the secondary resistance divided by the slip.

The equivalent circuit for a single-phase motor having a primary impedance of $R_{1M} + jX_{1M}$ and the primary winding of which is called the M phase is represented in Figure 2A(a).

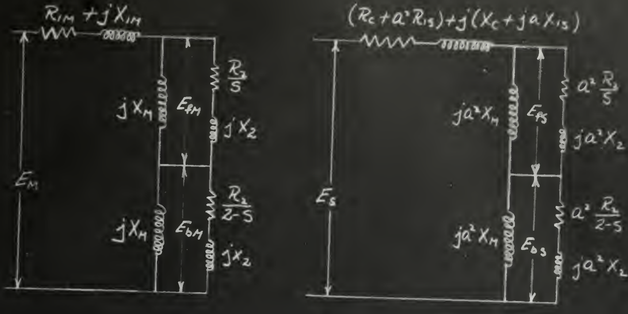
In this equivalent circuit the impressed voltage equivalent to the voltage induced in phase M by the forward revolving field of flux is E_{fM} and the impressed voltage equivalent to the voltage generated by the backward revolving field of flux is E_{bM}.

If instead of the M phase there is an S phase displaced backward in position by 90 degrees, electrical,

from the position of the M phase and having a times as many turns as the M phase, the single-phase equivalent circuit for the S phase is shown in Figure 2A(b). In the S equivalent circuit the primary impedance is $a^2 R_{1s} + ja^2 X_{1s}$ and the external series impedance is $R_c + jX_c$.

If both the M and the S phases exist on the motor and are excited simultaneously, their fluxes superimpose without distortion and the equivalent circuits are the same as before except that in addition to the forward and backward voltages self induced in each phase, there is a forward and backward voltage due to the fluxes of the other phase. Under this condition the equivalent circuit becomes as shown in Figure 2B wherein the divided circuits have been replaced by series impedances of equal value.

Since the M phase is displaced forward by 90 electrical degrees from the S phase, the voltage generated in the M phase by the S forward flux must lag by 90 time degrees the voltage which the same flux produces in the S phase and since the turns on the M phase are to those on the S phase as 1 is to a, the magnitude of the M voltage must be 1/a times that of the S phase. From these considerations the equation for the impressed voltage equivalent to the



(a) (b)

Fig. 2A - Single-phase motor equivalent circuits.

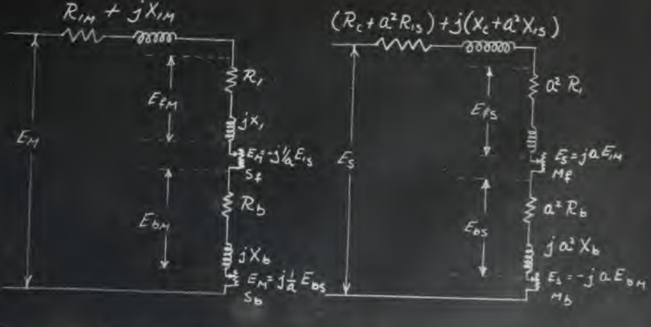


Fig. 2B - Unbalanced two phase motor equivalent circuits.

voltage produced in the M phase by the S forward flux is:

$$\underset{sf}{E}_M = -j\frac{1}{a}E_{fs} \tag{1}$$

By a similar line of reasoning the impressed voltages equivalent to the other mutually generated voltages can be shown to be the values given on the equivalent circuit.

The equations for the voltages impressed on each of the primary phases are from Figure 2B.

$$\begin{aligned} E_M = {} & I_M\left[(R_{1M}+R_f+R_b)+j(X_{1M}+X_f+X_b)\right] \\ & -jI_S\,a\left[(R_f-R_b)+j(X_f-X_b)\right] \end{aligned} \tag{2}$$

$$\begin{aligned} E_S = {} & I_S\left[R_c+a^2(R_{1S}+R_f+R_b)+jX_c+ja^2(X_{1S}+X_f+X_b)\right] \\ & +jI_M\,a\left[(R_f-R_b)+j(X_f-X_b)\right] \end{aligned} \tag{3}$$

The simultaneous solution of (2) and (3) gives for the currents of an unbalanced two-phase motor:

$$\begin{aligned} I_M = {} & \frac{E_M\left\{\left[R_c+a^2(R_{1S}+R_f+R_b)\right]+j\left[X_c+a^2(X_{1S}+X_f+X_b)\right]\right\}}{\left\{\left[R_c+a^2(R_{1S}+R_f+R_b)\right]+j\left[X_c+a^2(X_{1S}+X_f+X_b)\right]\right\}} \\ & \frac{+\;jE_S\,a\left[(R_f-R_b)+j(X_f-X_b)\right]}{\left\{\left[R_b+R_f+R_{1M}\right]+j\left[X_{1M}+X_f+X_b\right]\right\}-a^2\left[(R_f-R_b)+j(X_f-X_b)\right]^2} \end{aligned} \tag{4}$$

$$\begin{aligned} I_S = {} & \frac{E_S\left[(R_{1M}+R_f+R_b)+j(X_{1M}+X_f+X_b)\right]-jE_M a\left[(R_f-R_b)+j(X_f-X_b)\right]}{\left\{\left[R_c+a^2(R_{1S}+R_f+R_b)\right]+j\left[X_c+a^2(X_{1S}+X_f+X_b)\right]\right\}} \\ & \cdot\left\{\left[R_{1M}+R_f+R_b\right]+j\left[X_{1M}+X_f+X_b\right]\right\}-a^2\left[(R_f-R_b)+j(X_f-X_b)\right]^2 \end{aligned} \tag{5}$$

Since only current and flux waves traveling at the

same speed can act together to produce average torque, the average torque in synchronous watts produced by an unbalanced two-phase motor is equal to the sum of the watts consumed by the forward field in each phase minus the sum of the watts consumed by the backwardfield.

If the current in the M phase be:

$$I_M = A + jB$$

and that in the S phase be

$$I_S = g + jh.$$

The equation for the average torque can be shown to be:

$$T_{avg} = (I_M^2 + a^2 I_S^2)(R_f - R_b) + 2a(Ah - Bg)(R_f + R_b). \qquad (6)$$

In addition to the average torque of Equation (6) there is an alternating torque produced through the action of the forward flux and backward current and vice-versa.

The maximum value of the alternating torque is:

$$T_{\widetilde{Max}} = \sqrt{(I_M^4 + a^4 I_S^4 + 2 I_M^2 I_S^2 a^2 \cos 2\emptyset)\left[(R_f - R_b)^2 + (X_f - X_b)^2\right]}. \qquad (7)$$

It will be noticed that the alternating torque ceases to exist when $I_S a$ is equal to I_M and $\emptyset$ is 90 degrees.

For purposes of analysis it is possible to represent either the capacitor or the capacitor transformer unit of the two-value capacitor motor by a resistance in series with a condensive reactance. If these values be substituted for the external impedance $R_c + jX_c$ in the

unbalanced two-phase equations and the voltage E_M be substituted for E_s the unbalanced two-phase equations apply directly to a capacitor motor.

The unbalanced two-phase torque equations are not affected by the above substitutions and apply without change to a capacitor motor.

The equation for the net capacitor motor output is:

$$W.O. = \left\{\left[I_M^2 + a^2 I_s^2\right]\left[R_f - R_b\right] + 2 I_M I_s a \left[R_f + R_b\right] \sin\phi\right\}(1-s) - W_f . \quad (8)$$

Since $Ah - Bg = I_M I_s \sin\phi$ wherein W_f is the watts friction loss.

The total current I of a capacitor motor is equal to the sum of I_M and I_s and if it be represented as follows:

$$I = C + jF \quad (9)$$

the equation for the input may be written:

$$W.I. = C\,E_M + W_i \quad (10)$$

in which W_i is the iron loss.

At standstill the torque Equation (6) becomes

$$T_{ss} = 2a(Ah - gB)(R_f + R_b). \quad (11)$$

Tests have been run on a capacitor motor to test the current equations of the revolving field theory. It was noted that the calculations and tests checked remarkably well.

The torque and performance characteristics of the tested motor under running conditions are given in

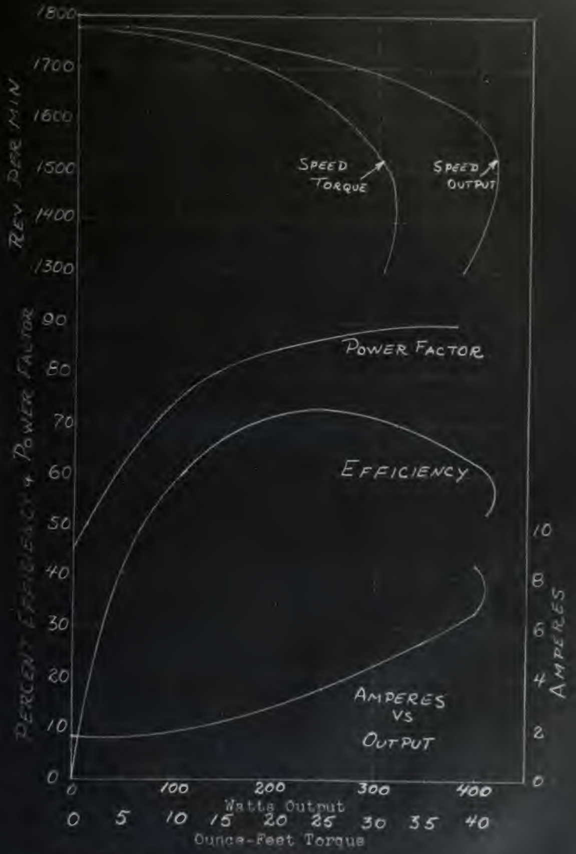


Fig. 2C - Performance Characteristics of ½ H.P. capacitor motor. (constants with Fig. 2D)

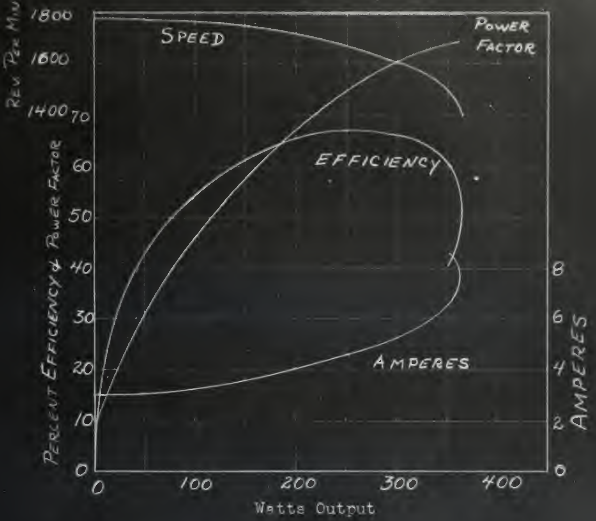


Fig. 2D - Single phase performance of $\frac{1}{4}$ H.P. capacitor motor without capacitor.

Constants

$R_{1M} = 2.02$

$R_2 = 2.06$

$R_{1S} = 5.12$

$R_c = 9$

$X_{1M} = 2.79$

$X_2 = 1.06$

$X_{1S} = 2.31$

$X_c = -172$

$X_M = 33.4$

$a = 1.18$

$W_f = 13$

$W_i = 24$

Figure 2C. For purposes of comparison, Figure 2D is inserted to show the calculated performance of the same motor with the S phase disconnected and operated as a single phase induction motor.

NOMENCLATURE FOR REVOLVING FIELD THEORY

A = "In phase" component of M phase current

a = Ratio $\frac{\text{Fundamental conductors on S phase}}{\text{Fundamental conductors on M phase}}$

$a^2 R_{1s}$ = Primary resistance of S phase

$a^2 X_{1s}$ = Primary leakage reactance of S phase

B = "Reactive" component of M phase current

C = "In phase" component of total current

E_M = Vector voltage impressed on primary of M phase

E_S = Vector voltage impressed on primary of S phase

F = "Reactive"component of total current

h = "Reactive" component of S phase current

$\dot{I}$ = Vector total current

I = Effective value of total current

$\dot{I}_M$ = Vector M phase current

I_M = Effective value of M phase current

$\dot{I}_s$ = Vector S phase current

I_S = Effective value of S phase current

g = In-phase component of S phase current

R_{1M} = Primary resistance of M phase

R_2 = Secondary resistance reduced to M phase

R_c = External resistance in series with S phase

R_b = Apparent resistance to M phase backward field

R_f = Apparent resistance to M phase forward field

T_{avg} = Average value of developed torque

T_{ss} = Standstill torque

$T_{\widetilde{MAX}}$ = Maximum value of alternating torque

W_f = Watts friction and windage loss

W_i = Watts fundamental iron loss

W.I. = Total watts input

W.O. = Net watts output

X_{1M} = Primary leakage reactance of M phase

X_2 = Secondary leakage reactance reduced to M phase

X_c = External reactance in series with S phase

X_b = Apparent reactance to M phase backward field

X_f = Apparent reactance to M phase forward field

X_m = Magnetizing reactance of M phase

ϕ = Angle by which $\dot{I}_S$ leads $\dot{I}_M$

CHAPTER III

CALCULATION SHEET (11)

The revolving field theory of the capacitor motor was presented in the previous chapter giving the complete equations for calculating the performance and torque of a capacitor motor at any value of slip. The equations are straightforward, but involve vector operations and are rather slow for design work. Mr. P. H. Trickey developed a calculation sheet for using these formulas, where all the operations except the last in calculating the currents are performed without vectors.

First he rewrote the three fundamental equations 4, 5, and 6 of the previous chapter using the symbols of Table I.

TABLE I

New Symbol	Old Symbol	Definition
K	a	$\frac{\text{Tot.effective series conductors in aux. phase}}{\text{Tot.effective series conductors in main phase}}$
r_{1a}	a^2R_{1s}	Primary resistance of auxiliary phase
x_{1a}	a^2X_{1a}	Primary leakage reactance of auxiliary phase
r_{1m}	R_{1m}	Primary resistance of main phase
r_2	$2R_2$	Total seconds resistance referred to primary of main phase
x_{1m}	X_{1m}	Primary leakage reactance of main phase
x_2	$2X_2$	Total seconds leakage reactance referred to primary of main phase
X_m	$2X_m$	Total magnetizing reactance of main phase
s	S	Slip in per unit of synchronism
I_a	I_s	Auxiliary phase current

These equations with the new symbols become

$$I_M = E_M\left\{\frac{[R_c + r_{1a} + K^2(R_f+R_b)] + j[X_c + x_{1a} + K^2(X_f+X_b)]}{\{[R_c+r_{1a}+K^2(R_f+R_b)] + j[X_c+x_{1a}+K^2(X_f+X_b)]\}}\right. \qquad (1)$$

$$\left.\frac{+jK[(R_f-R_b)+j(X_f-X_b)]}{\{(r_{1m}+R_f+R_b)+j(x_{1m}+X_f+X_b)\}-K^2[(R_f-R_b)+j(X_f-X_b)]^2}\right\}$$

$$I_a = E_m\left\{\frac{[(r_{1m}+R_f+R_b)+j(x_{1m}+X_f+X_b)] - jK[(R_f-R_b)+j(X_f-X_b)]}{\begin{array}{c}\{[R_c+r_{1a}+K^2(R_f+R_b)]+j[X_c+X_{1a}+K^2(X_f+X_b)]\}\\ \{(r_{1m}+R_f+R_b)+j(x_{1m}+X_f+X_b)\}-K^2[(R_f-R_b)+j(X_f-X_b)]^2\end{array}}\right\} \qquad (2)$$

$$T_{avg} = (I_m^2 + K^2 I_a^2)(R_f - R_b) + 2K(Ah - Bg)(R_f + R_b). \qquad (3)$$

Next, the forward and backward impedances were rewritten with the new symbols. For the revolving field theory, Mr. Morrill showed that

$$R_f = \frac{X_m^2 R_2 \div S}{(R_2/S)^2 + (X_2+X_m)^2}$$

This equation with the new notation becomes

$$R_f = \frac{(X_m \div 2)^2 (r_2 \div 2)(1 \div s)}{(r_2 \div 2)^2 (1 \div s) + \left(\frac{x_2 + X_m}{2}\right)^2} .$$

Arbitrarily the following definitions are substituted into the equation and the numerator and denominator are divided by $X_m^2/4$.

$$X_o = X_m + x_2, \quad K_p = X_m \div (X_m + x_2)$$

$$K_r = K_p^2 = X_m^2 \div X_o^2 .$$

Thus

$$R_f = \frac{1}{2} \frac{(r_2 K_r)(1 \div s)}{[(r_2 \div X_o)(1 \div s)]^2 + 1} .$$

R_b becomes the same equation with (2-s) in place of (s). Mr. Morrill's reactance term for the forward reactance is

$$X_f = \frac{X_m[(R_2 \div S)^2 + X_2(X_2 + X_m)]}{(R_2 \div S)^2 + (X_2 + X_m)^2} .$$

This equation rewritten in the new symbols and using X_o and K_p and dividing the numerator and denominator by $X_o^2/4$ results in

$$X_f = \frac{1x_2K_p + (r_2 K_p)(r_2/X_o)(1/s)^2}{2 \quad [(r_2/X_o)(1/s)]^2 + 1}$$

and

$$X_b = \frac{1}{2} \frac{x_2K_p + (r_2 K_p)(r_2/X_o)(1/(2-s))^2}{[(r_2/X_o)(1/2-s))]^2 + 1} .$$

In the equations for I_m and I_a, there are many groups of terms that are the same. Rewriting the equations, using as symbols the item numbers of the final calculation sheet gives,

$$I_m = A + jB = E_m x$$

$$\left\{ \frac{(24) + j(124) + jK[(15) + (115)]}{[(24) + j(124)][(18) + j(118)] - K^2[(15) + j(115)]^2} \right\}$$

$$I_a = g + jh = E_m x \left\{ \frac{(18)+j(118)-jK\left[(15)+j(115)\right]}{\left[(24)+j(124)\right]\left[(18)+j(118)\right]-K^2\left[(15)+j(115)\right]^2} \right\}$$

Simplifying to obtain one real and one quadrature term in both numerator and denominator,

$$A + jB = E_m x \left\{ \frac{\left[(24)-K(115)\right]+j\left[(124)+K(15)\right]}{\left[(24)(18)-(124)(118)-K^2(15)^2+K^2(115)^2\right] + j\left[(18)(124)+(118)(24)-2(K15)(K115)\right]} \right\}$$

$$g + jh = E_m x \left\{ \frac{\left[(18)-K(115)\right]+j\left[(118)-K(15)\right]}{\left[(24)(18)-(124)(118)-K^2(15)^2+K^2(115)^2\right] + j\left[(18)(124)+(118)(24)-2(K15)(K115)\right]} \right\}.$$

Assigning number symbols to the above groups of terms,

$$A + jB = E_m \frac{(26)+j(126)}{(32)+j(132)}$$

and

$$g + jh = E_m \frac{(20)+j(120)}{(32)+j(132)} .$$

Dividing the numerator and denominator by E_m gives the following result

$$A + jB = \frac{(26)+j(126)}{(33)+j(133)}$$

and

$$g + jh = \frac{(20)+j(120)}{(33)+j(133)} .$$

These last two equations are the only ones where vector operation is necessary. As shown in Figure 3A the calculation sheet is laid out so that it is a simple matter to get the currents of the motor.

The torque equation can be calculated without any vector operations, which is particularly desirable in calculating speed torque curves.

In order to avoid making the vector operation involved in obtaining A, B, g, and h of equation (3) let

$$N_m = \sqrt{(26)^2 + (126)^2}$$

$$N_a = \sqrt{(20)^2 + (120)^2}$$

$$V = \sqrt{(33)^2 + (133)^2}$$

Also since (Ah - Bg) is the quadrature term obtained by multiplying (g + jh) by the conjugate of (A + jB) performing this operation on both right and left sides of the equations gives an expression for (Ah - Bg).

$$(A - jB)(g + jh) = (Ag + Bh) + j(Ah - Bg)$$

$$= \frac{(26) - j(126)}{(33) - j(133)} \cdot \frac{(20) + j(120)}{(33) + j(133)}$$

$$\frac{[(26)(20) + (126)(120)] - j[(26)(120) - (126)(20)]}{(33)^2 + (133)^2}.$$

Thus

$$Ah - Bg = \frac{(26)(120) - (126)(20)}{V^2}$$

and

$$T = \left[(N_m/V)^2 + (KN_a/V)^2\right](R_f - R_b) + 2K\left[\frac{(26)(120)-(126)(20)}{V^2}\right](R_f + R_b).$$

Mr. P. H. Trickey used a slightly different method of accounting for iron loss than had Mr. Morrill. Mr. Morrill's equations neglect iron loss and then correct for it by adding it to the friction and windage losses as a reduction in output.

The vector operations of items 35, and 37 give fictitious values of current which have been obtained while neglecting iron loss entirely. These fictitious values are used in calculating the internal torque and output. The friction, windage, and odd-frequency iron losses are then subtracted to give the useful output. A close approximation to the true current is obtained by adding to the in-phase component of the main winding current a value of in phase current obtained by dividing the fundamental-frequency iron loss by the voltage. Thus the final main phase current is then

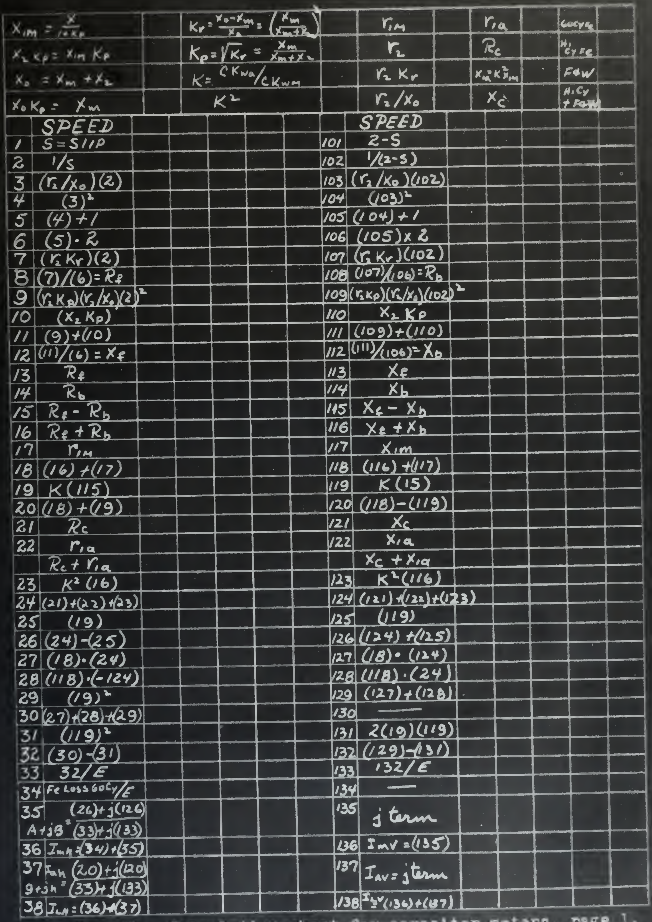
$$A + \frac{\text{fundamental frequency iron loss}}{E} + jB.$$

Mr. Trickey has had considerable success in approximating the true conditions with the above reasoning. It seemed to agree with test results much closer than by adding half to the main phase and half to the auxiliary

phase. The auxiliary phase current is $g + jh$ and the line current is the vector sum.

This method accounting for the iron loss is based on the fact that the fundamental frequency iron losses are normally supplied by the stator and the odd frequency losses of the surfaces and tooth pulsations are supplied by the rotating member with a corresponding loss of torque.

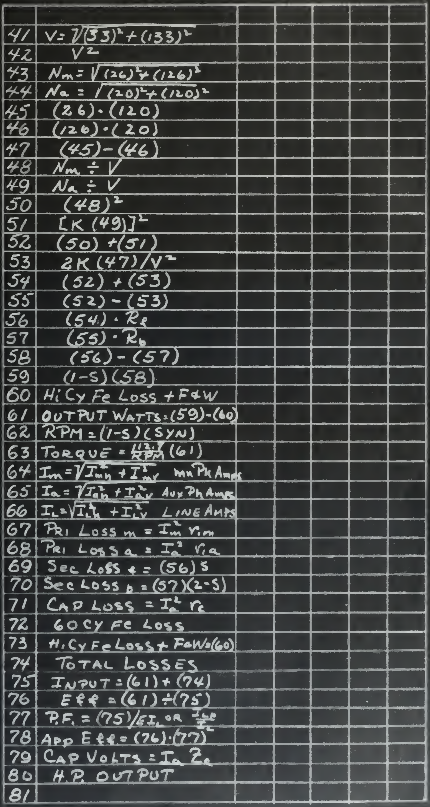
In Figures 3A and 3B, the fundamental-frequency iron loss is denoted by the symbol "60 Cy Fe" and the odd frequency iron loss by the symbol "Hi Cy Fe." F + W is used as the symbol for friction and windage.



$X_{1m} = \frac{X}{1+K_p}$		$K_r = \frac{X_0 - X_m}{X_2} = \left(\frac{X_m}{X_m+X_2}\right)$		r_{1M}		r_{1a}		60cy Fe	
$X_2 K_p = X_{1m} K_p$		$K_p = \sqrt{K_r} = \frac{X_m}{X_m+X_2}$		r_2		R_c		H1 Cy Fe	
$X_0 = X_m + X_2$		$K = CKwa/CKwm$		$r_2 K_r$		$X_{1a} = K^2 X_{1m}$		F&W	
$X_0 K_p = X_m$		K^2		r_2/X_0		X_C		H1 Cy + F&W	

	SPEED							SPEED					
1	S = SLIP						101	2 − S					
2	1/S						102	1/(2 − S)					
3	$(r_2/X_0)(2)$						103	$(r_2/X_0)(102)$					
4	$(3)^2$						104	$(103)^2$					
5	(4) + 1						105	(104) + 1					
6	(5) · 2						106	(105) x 2					
7	$(r_2 K_r)(2)$						107	$(r_2 K_r)(102)$					
8	$(7)/(6) = R_f$						108	$(107)/(106) = R_b$					
9	$(r_2 K_p)(r_2/X_0)(2)^2$						109	$(r_2 K_p)(r_2/X_0)(102)^2$					
10	$(X_2 K_p)$						110	$X_2 K_p$					
11	(9) + (10)						111	(109) + (110)					
12	$(11)/(6) = X_f$						112	$(111)/(106) = X_b$					
13	R_f						113	X_f					
14	R_b						114	X_b					
15	$R_f - R_b$						115	$X_f - X_b$					
16	$R_f + R_b$						116	$X_f + X_b$					
17	r_{1M}						117	X_{1m}					
18	(16) + (17)						118	(116) + (117)					
19	K(115)						119	K(15)					
20	(18) + (19)						120	(118) − (119)					
21	R_c						121	X_c					
22	r_{1a}						122	X_{1a}					
	$R_c + r_{1a}$							$X_c + X_{1a}$					
23	$K^2(16)$						123	$K^2(116)$					
24	(21) + (22) + (23)						124	(121) + (122) + (123)					
25	(19)						125	(119)					
26	(24) − (25)						126	(124) + (125)					
27	(18) · (24)						127	(18) · (124)					
28	(118) · (−124)						128	(118) · (24)					
29	$(19)^2$						129	(127) + (128)					
30	(27) + (28) + (29)						130	—					
31	$(119)^2$						131	2(19)(119)					
32	(30) − (31)						132	(129) − (131)					
33	32/E						133	132/E					
34	Fe Loss 60Cy/E						134	—					
35	(26) + j(126)						135	j term					
	A + jB = (33) + j(133)												
36	I_{mh} = (34) + (35)						136	I_{mv} = (135)					
37	I_{ah} (20) + j(120)						137	I_{av} = j term					
	g + jh = (33) + j(133)												
38	I_{Lh} = (36) + (37)						138	I_{Lv} = (136) + (137)					

Fig. 3A - Calculation sheet for capacitor motors, page 1.



41	$V = \sqrt{(33)^2 + (133)^2}$					
42	V^2					
43	$N_m = \sqrt{(26)^2 + (126)^2}$					
44	$N_a = \sqrt{(20)^2 + (120)^2}$					
45	$(26)\cdot(120)$					
46	$(126)\cdot(20)$					
47	$(45) - (46)$					
48	$N_m \div V$					
49	$N_a \div V$					
50	$(48)^2$					
51	$[K\,(49)]^2$					
52	$(50) + (51)$					
53	$2K\,(47)/V^2$					
54	$(52) + (53)$					
55	$(52) - (53)$					
56	$(54)\cdot R_f$					
57	$(55)\cdot R_b$					
58	$(56) - (57)$					
59	$(1-S)(58)$					
60	Hi Cy Fe Loss + F&W					
61	OUTPUT WATTS = (59) - (60)					
62	RPM = (1-S)(SYN)					
63	TORQUE = $\frac{112.7}{RPM}(61)$					
64	$I_m = \sqrt{I_{mh}^2 + I_{mv}^2}$ Mn Ph Amps					
65	$I_a = \sqrt{I_{ah}^2 + I_{av}^2}$ Aux Ph Amps					
66	$I_L = \sqrt{I_{Lh}^2 + I_{Lv}^2}$ LINE AMPS					
67	PRI LOSS $_m = I_m^2\, r_{1m}$					
68	PRI LOSS $_a = I_a^2\, r_{1a}$					
69	SEC LOSS $_f = (56)\,S$					
70	SEC LOSS $_b = (57)(2-S)$					
71	CAP LOSS $= I_a^2\, r_c$					
72	60 CY FE LOSS					
73	Hi Cy Fe Loss + F&W = (60)					
74	TOTAL LOSSES					
75	INPUT = (61) + (74)					
76	Eff = (61) ÷ (75)					
77	P.F. = $(75)/EI_L$ OR $\frac{I_{Lh}}{I_L}$					
78	App Eff = (76)·(77)					
79	CAP VOLTS = $I_a Z_c$					
80	H.P. OUTPUT					
81						

Fig. 3B - Calculation sheet for capacitor motors, page 2.

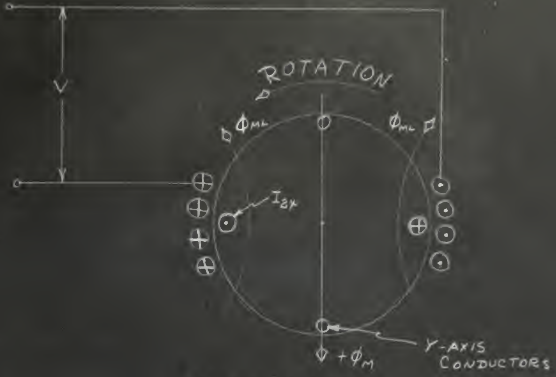
CHAPTER IV

THE CROSS FIELD THEORY(8)

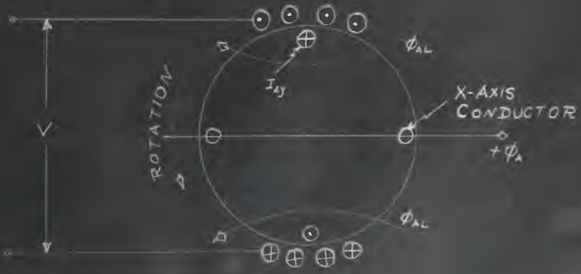
The main winding of the capacitor motor produces reaction similar to the transformer action of the simple phase action. That is the alternating current flowing through the main windings will set up a pulsating magnetic field. This pulsating field will induce a voltage in the secondary winding (rotor) by transformer action, and since the secondary winding is short-circuited a current will flow. The auxiliary winding will produce a magnetic flux that is in space quadrature with the flux of the main winding and is generally similar to the speed flux of the simple induction motor. The rotor is generally of the squirrel-cage type. The reactions of the squirrel-cage rotor can be divided into two axes, namely, the X-axis and the Y-axis.

The two speed voltages of the main-winding circuit are produced by the X-axis conductors cutting the auxiliary-winding mutual flux and the Y-axis rotor leakage flux. Similarly the two speed voltages in the auxiliary circuit are due to the Y-axis conductors cutting the main-winding mutual flux and the X-axis rotor leakage flux. This feature is shown in Figure 4A.

If the motor is treated as two coupled circuits, it would be natural to assume that four voltage equations



(a) Main winding



(b) Auxiliary winding

Fig. 4A - Mutual and rotor leakage fluxes of the capacitor motor.

are necessary to define completely the operation of these two circuits. Before stating these equations it is well to describe the notation that is used.

The two windings on the stator are assumed to be in space quadrature. The winding having the oil consdenser in series is designated as the auxiliary winding and is used continuously. The subscript **A** will be used for terms pertaining to this winding contrasted to M for the main winding. The rotor bars will be assumed to generate speed voltages by cutting both rotor leakage and mutual or air gap fluxes (as the rotor turns, its bars cut through the main axis flux, generating a voltage by speed action).

All rotor values are given in main-winding stator terms. Usually there are more turns on the auxiliary winding than on the main, so that rotor constants in auxiliary winding terms would be higher (or at least different). The ratio is

$$a = \frac{\text{effective auxiliary winding turns}}{\text{effective main winding turns}} .$$

Now in dealing with rotor currents which will be operative in the auxiliary axis, they should be multiplied by 'a' before combining with other values. Similarly, any magnetizing reactance reflected to the rotor, operative in the auxiliary axis, should be divided by 'a' if it is necessary to combine its effect with main-axis terms. This does not hold for the rotor impedance

since its values are in main winding terms throughout.

With these preliminary statements, we can write the fundamental equations.

(a) The voltage applied to the main winding overcomes a local stator impedance drop and the counter emf built up by the mutual flux of the main axis.

The counter emf will be the reference vector. It is numerically,

$$E_M = I_{\phi M} Z_M \tag{2}$$

If the core loss is neglected, Z_{MM} reduces to jX_{MM}, where jX_{MM} = the magnitizing reactance of the main winding.

(b) The load will require a current component in the main or X-axis which is designated I'_x. Reflected to the stator, this will be I_x, and the total stator current of the main winding is the vector sum of this current and the "exciting branch current,"

$$I_M = I_x + I_{\phi M} \tag{3}$$

or

$$I_{\phi M} = I_M - I_x \tag{4}$$

combining (2) and (4)

$$E_M = (I_M - I_x) Z_{MM} \tag{5}$$

The equation for the condition expressed in item (a) is then

$$V = Z_{MM}(I_M - I_x) + I_M Z_{1M}. \tag{6}$$

(c) The voltage applied to the auxiliary winding overcomes the local stator impedance drop, the reactance

or impedance of the condenser and the counter emf built up by the mutual flux of the cross or auxiliary axis.

The counter emf will be, numerically,

$$E_A = I_{\phi A} Z_{MA} \tag{7}$$

neglecting the core loss, Z_{MA} reduces to jX_{MA}.

(d) The stator current of the auxiliary winding is the resultant of a load component (I_{2y}) reflected by transformer action from the rotor, and an auxiliary-axis magnitizing current similar to that of the main winding but usually different in magnitude.

$$I_A = I_y + I_{\phi A} \tag{8}$$

$$I_{\phi A} = I_A - I_y \tag{9}$$

Combining equations (7) and (9)

$$E_A = (I_A - I_y)Z_{MA} \tag{10}$$

The equation for the condition expressed in item c is then

$$V = Z_{MA}(I_A - I_y) + I_A (Z_{1A} + Z_c) \tag{11}$$

If the resistance of the condenser impedance is neglected, it becomes

$$-jX_c = \frac{-j10}{2\pi\, f \text{x} \text{microfarads}} \tag{12}$$

(e) In the rotor, the main or X-axis current results from two emf's.

(1) The transformer voltage opposite in phase to the stator counter emf, but of the same magnitude with the rotor constants in main winding terms,

$Z_{MM}(I_M - I_x)$.

(2) A speed voltage, resulting from the rotor bars cutting the cross-axis mutual flux and the rotor cross-axis leakage flux. A speed voltage is K times as large as the transformer voltage caused by the same flux.

Speed voltage, cross-axis mutual flux;

$$K\, Z_{MA}(I_A - I_y) \tag{13}$$

To combine with main or X-axis terms

$$\frac{K\, Z_{MA}}{a^2}(I_A - I_y)\ a = \frac{K\, Z_{MA}}{a}(I_A - I_y) \tag{14}$$

(When the rotor revolves through the air-gap flux, its conductors cut the flux lines, generating an emf by speed action. Rotation of a conductor through a flux produce (K) times the voltage produced by pulsation of that flux;

$$\text{where } K = \frac{\text{actual rotor rpm}}{\text{synchronous rpm}}\ .$$

Speed voltage, leakage flux, in stator terms:

$$a\,(I_y\, X_2) \tag{15}$$

Combining, and considering that a speed emf reaches its maximum at the same time as the flux causing it,

$$\text{Final speed emf} = K\left[\frac{j\ Z_{MA}}{a}(I_A - I_y) + a\ I_y X_2\right] \tag{16}$$

By Kirchoff's law the vector sum of the voltages of item e1, e2 and the rotor impedance drop caused by the main-axis current equals zero.

$$Z_{MM}(I_M - I_X) + K\left[\frac{jZ_{MA}}{a}(I_A - I_Y) + a\, I_Y X_2\right] + I'_X Z_2 = 0 \tag{17}$$

or

$$Z_{MM}(I_M - I_Y) + \left[K\,\frac{jZ_{MA}}{a}(I_A - I_Y) + a\, I_Y X_2\right] - I_X Z_2 = 0 \tag{18}$$

(f) In the auxiliary or Y-axis of the rotor, two emf's similar to those of item (e) are present.

(1) A transformer voltage opposite in phase to the cross-axis counter emf, but different in magnitude by the ratio of transformation.

$$\frac{Z_{MA}}{a^2}(I_A - I_Y)\, a = \frac{Z_{MA}}{a}(I_A - I_Y) \tag{19}$$

(2) A speed voltage resulting from the rotor bars' cutting the main winding mutual and rotor-leakage fluxes. From the mutual flux, the emf from speed action is

$$K\, Z_{MM}(I_M - I_X) \tag{20}$$

For the leakage flux, in stator terms

$$K\, I_X X_2 \tag{21}$$

Combining and locating the vector position,

$$K\left[-j\, Z_{MM}(I_M - I_X) - I_X X_2\right] \tag{22}$$

By Kirchoff's law the vector sum of these voltages and the rotor impedance drop of the cross-axis rotor current equals zero.

$$\frac{Z_{MA}}{a}(I_A - I_Y) + K\left[-j\, Z_{MM}(I_M - I_X) - I_X X_2\right] + I_{2X} Z_2 = 0 \tag{23}$$

or

$$\frac{Z_{MA}}{a}(I_A - I_Y) + K\left[-j\, Z_{MM}(I_M - I_X) - I_X X_2\right] - a I_X Z_2 = 0 \tag{24}$$

The four fundamental equations are 6, 11, 18, and 24 involving four currents I_M, I_A, I_x, and I_y. They must be solved simultaneously.

These equations are extensions of West's equations for the cross-field theory applied to the plain single phase induction motor. The terms $(I_x X_2)$ and $(a\, I_y X_2)$ in equations 24 and 18, respectively, represent the added effect on the speed electromotive forces of cutting the rotor leakage fluxes. If these are dropped, the fundamental equations reduce to a form which Steinmetz's single-phase motor equations would yield.

By substitution, the stator components of the rotor currents, I_x, and I_y, can be eliminated in these four equations leaving two equations in terms of the two components of stator current.

The two currents I_M and I_A in the two remaining equations can be expressed in the following form:

$$A_1 I_M + B_1 I_A = C_1$$

$$A_2 I_M + B_2 I_A = C_2$$

The expressions for the coefficients represented by the constants A, B, and C in the above equations can be found at the end of this chapter. Then by determinants:

$$I_M = \frac{C_1 B_2 - C_2 B_1}{A_1 B_2 - A_2 B_1}$$

$$I_A = \frac{A_1 C_2 - A_2 C_1}{A_1 B_2 - A_2 B_1}$$

From equations (6) and (11),

$$I_x = I_M\left(1 + \frac{Z_{1M}}{Z_{MM}}\right) - \frac{V}{Z_{MM}}$$

$$I_y = I_A\left(\frac{Z_{MM} + Z_{1A} + Z_c}{Z_{MA}}\right) - \frac{V}{Z_{MA}}$$

When the motor is stationary, the term K reduces to zero and the currents become

$$I_M = \frac{V}{Z_{1M} + \left(\dfrac{Z_{2M}}{1 + \dfrac{Z_{2M}}{Z_{MM}}}\right)}$$

$$I_A = \frac{V}{(Z_{1A} + Z_c) + \dfrac{aZ_{2M}}{\dfrac{1}{a} + \dfrac{aZ_{2M}}{Z_{MA}}}}$$

The counter electromotive forces in the separate windings are:

$$E_M = V - I_M Z_{1M}$$

$$E_A = V - I_A(Z_{1A} + Z_c)$$

The rotor input, or the power transferred across the gap, is the sum:

$$E_M I_x \cos(E_M, I_x) + E_A I_y \cos(E_A, I_y) \qquad (25)$$

The power converted to mechanical form:

Power across gap by (25) - $(I_x^2 + a^2 I_y^2) r_2 = ST$

Net output = power converted to mechanical form - (F & W)

The torque in synchronous watts, (T) is the sum:

$$aE_M I_y \sin(E_M, I_y) + \frac{E_A}{a} I_x \sin(E_A, I_x)$$

The stator input is the sum:

$$V\ I_M \cos\theta_M + V\ I_A \cos\theta_A$$

The stator losses:

$I_M^2 r_{1M}$ = copper loss in the main winding

$I_A^2 (r_{1s} + r_c)$ = copper loss in auxiliary winding circuit

$\frac{E_M^2}{r_0} + \frac{E_A^2}{r_{0A}}$ = iron losses

The rotor losses

$I_x^2 r_2$ = copper loss in the x-axis

$a^2 I_y^2 r_2$ = copper loss in the y-axis

Expressions for the constants A, B, and C.

$$A_1 = K\left[jZ_{1M} - X_{2M}(1 + Z_{1M}/Z_{MM})\right]$$

$$B_1 = -\left[aZ_{2M} + (Z_{1A} + Z_c)(1/a + aZ_{2M}/Z_{MA})\right]$$

$$C_1 = V\left[K(j - X_{2M}/Z_{MM}) - (1/a + aZ_{2M}/Z_{MA})\right]$$

$$A_2 = \left[Z_{1M}(1 + Z_{2M}/Z_{MM}) + Z_{2M}\right]$$

$$B_2 = K\left[-aX_{2M} + (Z_{1A} + Z_c)(j/a - aX_{2M}/Z_{MA})\right]$$

$$C_2 = V\left[(1 + Z_{2M}/Z_{MM}) + K(j/a - aX_{2M}/Z_{MA})\right]$$

CROSS FIELD THEORY SYMBOLS AND NOTATION

V - Applied stator voltage

E_M - The counter electromotive force in the main axis

E_A - The counter electromotive force of the auxiliary or capacitor winding

Z_{1M} - The impedance (local) of the main stator winding

Z_{1A} - The impedance (local) of the auxiliary stator winding

Z_C - Capacitor impedance

Z_{MM} - The magnetizing impedance in the M-axis

Z_{MA} - The magnetizing impedance in the A-axis

Z_{2M} - The rotor impedance in the main winding terms (local)

X_{2M} - rotor-circuit reactance in main winding terms

Z_{2A} - rotor-circuit impedance in auxiliary winding terms

X_{2A} - rotor-circuit reactance in auxiliary winding terms

I_M - stator current, main axis

$I_{\phi M}$ - magnetizing current, main axis

I_A - stator current, auxiliary axis

$I_{\phi A}$ - magnetizing current, auxiliary axis

I_x - stator component of the rotor current, main axis

I_y - stator component of the rotor current, auxiliary axis

I_{2x} - rotor current in X-axis (main winding)

I_{2y} - rotor current in Y-axis (auxiliary winding)

ϕ_M - main winding magnetic flux

ϕ_A - auxiliary winding magnetic flux

ϕ_{M1} - rotor, main winding leakage flux

ϕ_{A1} - rotor, auxiliary winding leakage flux

K - $\dfrac{\text{actual rpm}}{\text{synchronous rpm}}$

a - $\dfrac{\text{effective auxiliary winding turns}}{\text{effective main winding turns}}$

CHAPTER V

THE CIRCLE DIAGRAM(10)

The main purpose of this chapter is to present an operating circle diagram for the capacitor motor, showing that the input current vector follows a circular envelope, as the mechanical shaft power is varied.

The analyses which follow will begin with the fundamental cross field expressions as developed by Puchstein and Lloyd which are as follows:

$$V = Z_{MM}(I_M - I_x) + I_M Z_{1M} \tag{11a}$$

$$V = Z_{MA}(I_A - I_y) + I_A(Z_{1A} + Z_c) \tag{11b}$$

$$Z_{MM}(I_M - I_x) + \frac{K}{a}\left[jZ_{MA}(I_A - I_y) + a\, I_y X_{2M}\right] - I_x Z_{2M} = 0 \tag{11c}$$

$$Z_{MA}(I_A - I_y) - aK\left[jZ_{MM}(I_M - I_x) + I_x X_{2M}\right] - a\, Z_{2M} I_y = 0 \tag{11d}$$

Expressions of total input current will be obtained from equations 11, and these expressions will be seen to be similar to the basic expression of the circle diagram equation.

The circle diagram equation is

$$I_1 = \frac{C_s}{A_s} V + \frac{\dfrac{V}{A_s^2}}{\dfrac{B_s}{A_s} + Z_s} \tag{12}$$

The first step in this analysis is to solve the four basic equations 11 simultaneously, thereby eliminating the

the two rotor currents I_x and I_y. This will lead to two expressions as follows:

$$V\left(\frac{aZ_{SM}}{Z_{MM}}+\frac{jKZ'_{SA}}{Z_{MA}}\right)= jKI_A\left(\frac{Z'_{SA}Z_{1T}}{Z_{MA}}+jx_{2A}\right)+aI_M\left(Z_{1M}+\frac{Z_{SM}Z_{1M}}{Z_{MM}}\right) \quad (13a)$$

$$V\left(\frac{Z_{SA}}{Z_{MA}}-\frac{jaKZ'_{SM}}{Z_{MM}}\right)=I_A\left(Z_{2A}+\frac{Z_{SA}Z_{1T}}{Z_{MA}}\right)-jaKI_M\left(\frac{Z'_{SM}Z_{1M}}{Z_{MM}}+jx_{2M}\right) \quad (13b)$$

In the foregoing expressions as well as in later developments, it was found convenient to introduce a certain number of abbreviating terms. These are defined at the end of this chapter.

With the necessary substitutions, the two voltage equations 13 become

$$V(aD + jKD') = jKI_A Z'_A + aI_M Z_M \quad (14a)$$

$$V(D - jaKD') = I_A Z_A - jaKI_M Z'_M \quad (14b)$$

Solving these equations for I_M and I_A leads to the following:

$$I_M = \frac{V\left[(D\ Z_A - K^2 D' Z'_A) + jaKr_{2M}\right]}{Z_A Z_M - K^2 Z'_A Z'_M} \quad (15a)$$

$$I_A = \frac{V\left[(D\ Z_M - K^2 D' Z'_M) - jaKr_{2M}\right]}{Z_A Z_M - K^2 Z'_A Z'_M} \quad (15b)$$

The total current is equal to the sum of I_A and I_M.

$$I_1 = \frac{V\left[D\ (Z_A + Z_M) - K^2 D'(Z'_A + Z'_M)\right]}{Z_A Z_M - K^2 Z'_A Z'_M} \quad (16)$$

BLOCKED ROTOR CURRENTS OF CAPACITOR MOTOR

A special case of interest is that of blocked rotor.

The individual winding currents and the total current can be found by putting $K = N/N_S = 0$ and utilizing the identities listed at the end of this chapter.

Thus 15 and 16 become

$$I_{MB} = \frac{V\ D}{Z_M} \tag{17a}$$

$$I_{AB} = \frac{V\ D}{Z_A} \tag{17b}$$

$$I_{1B} = \frac{V\ D(Z_A + Z_M)}{Z_A Z_M} \tag{17c}$$

CAPACITOR MOTOR CURRENTS AT SYNCHRONOUS SPEED

A capacitor motor cannot operate at synchronous speed unless it is driven mechanically by some suitable prime mover. However, the current drawn by the motor under such conditions can be obtained by putting $K = N \div N_S = 1$ in equations 15 and 16, thus

$$I_{MS} = \frac{V\left[(D\ Z_A - D' Z'_A) + jar_{2M}\right]}{Z_A Z_M - Z'_A Z'_M} \tag{18a}$$

$$I_{AS} = \frac{V\left[(D\ Z_M - D' Z'_M) - jar_{2M}\right]}{Z_A Z_M - Z'_A Z'_M} \tag{18b}$$

$$I_{1S} = \frac{V\left[D\ (Z_A + Z_M) - D'\ (Z'_A + Z'_M)\right]}{Z_A Z_M - Z'_A Z'_M} \tag{18c}$$

OPEN-CIRCUIT CURRENTS

A third special case of equation 15 and 16 is that for infinite speed, which corresponds to the equivalent infinite load impedance.

Thus for $K = \text{infinity}$

$$I_{MO} = \frac{V\,D'}{Z'_M} \qquad (19a)$$

$$I_{AO} = \frac{V\,D'}{Z'_A} \qquad (19b)$$

$$I_{10} = \frac{V\,D'\,(Z'_A + Z'_M)}{Z'_A\,Z'_M} \qquad (19c)$$

CIRCLE DIAGRAM FOR SYNCHRONOUS SPEED, OR $K = 1$

The general equation for a circle diagram of a three phase induction motor can be shown to be equation 12 in which Z_s represents the equivalent circuit load resistance which replaces the mechanical output of the motor.

The subscript 's' is used in this section in order to distinguish the constants used here from similar values used later.

Rearranging equation (12) gives,

$$I_1 = \frac{C_s\,V}{A_s} + \frac{V}{A_s B_s + A_s^2 Z_s} \qquad (20)$$

This form of the equation is valuable in that it indicates that the total input current is equal to the sum of two currents: a constant current and a variable current. Furthermore, equation (20) suggests an equivalent circuit of two parallel branches having impedances

$$\frac{A_s}{C_s} \quad \text{and} \quad (A_s B_s + A_s^2 Z_s)$$

as illustrated by the circuit of Figure 5A(a).

Let the constant current I_{1S} of equation 20 represent the motor current at synchronous speed, namely equation 18c, thus

$$I_{1S} = \frac{C_S V}{A_S} = \frac{V\left[D\,(Z_A + Z_M) - D'(Z'_A + Z'_M)\right]}{Z_A Z_M - Z'_A Z'_M} \qquad (21)$$

Now, notice from Figure 5A(a) that the current through the equivalent load impedance Z_S is

$$I_{eS} = I_1 - I_{1S}$$

This may be applied to the actual motor circuit by subtracting equation 21 from equation 16, thus

$$I_{eS} = \frac{V\left[D(Z_A + Z_M) - K^2 D'(Z'_A + Z'_M)\right]}{Z_A Z_M - K^2 Z'_A Z'_M} - \frac{V\left[D(Z_A + Z_M) - D'(Z'_A + Z'_M)\right]}{Z_A Z_M - Z'_A Z'_M}$$

Simplifying this expression with

$$D' Z_M - D\,Z'_M = r_{2M}$$

and

$$D' Z_A - D\,Z'_A = a^2 r_{2M}$$

results in

$$I_{eS} = \frac{V}{\dfrac{(Z_A Z_M - Z'_A Z'_M) Z'_A Z'_M}{(Z_A Z'_A + a^2 Z_M Z'_M) r_{2M}} + \dfrac{(Z_A Z_M - Z'_A Z'_M)^2}{(Z_A Z'_A + a^2 Z_M Z'_M) r_{2M} (1 - K^2)}} \qquad (22)$$

Combining equations 21 and 22 results in an equation of the form of (20) which also complies with the equivalent circuit of Figure 5A(a), provided that

$$\frac{C_s}{A_s} = \frac{D(Z_A + Z_M) - D'(Z'_A + Z'_M)}{Z_A Z_M - Z'_A Z'_M} \tag{23a}$$

$$A_S B_S = \frac{Z'_A Z'_M (Z_A Z_M - Z'_A Z'_M)}{(Z_A Z'_A + a^2 Z_M Z'_M) r_{2M}} \tag{23b}$$

$$A_S^2 Z_S = \frac{(Z_A Z_M - Z'_A Z'_M)^2}{(Z_A Z'_A + a^2 Z_M Z'_M) r_{2M} - (1-K^2)} \tag{23c}$$

$$A_S = \frac{Z_A Z_M - Z'_A Z'_M}{Z_A Z'_A + a^2 Z_M Z'_M} \tag{23d}$$

$$Z_S = \frac{Z_A Z'_A + a^2 Z_M Z'_M}{(1-K^2) r_{2M}} \tag{23e}$$

from which

$$B_S = \frac{Z'_A Z'_M}{r_{2M}} \tag{23f}$$

$$C_S = \frac{D(Z_A + Z_M) - D'(Z'_A + Z'_M)}{Z_A Z'_A + a^2 Z_M Z'_M} \tag{23g}$$

By letting

$$Z_S = \frac{R_s}{1-K^2} \, /\overline{\phi_S} = \text{equivalent load impedance}$$

where

$$\frac{R_S}{1-K^2} = \text{absolute value of } Z_S$$

Equations 21 and 22 can be combined into a form similar to equation 12, thus

$$I_1 = \frac{C_s V}{A_s} + \frac{\frac{V}{A_s^2}\underline{/\phi_s}}{\frac{B_s}{A_s}\underline{/\phi_s} + \frac{R}{1-K^2}} \tag{24}$$

The term $(B_s \div A_s)\ \underline{/\phi_s}$ may be expressed in the form $r_s + jx_s$ from which the circle diameter is equal to

$$\frac{\frac{V}{A_s^2}\ \underline{/\phi_s}}{jx_s}$$

At synchronous speed, K will have a value of unity. The equivalent load impedance for such a value of K will be equal to infinity, as $Z_s = R/(1-K^2)$ and the circuit of Figure 5A(a) will degenerate to a single open circuit impedance of A_s/C_s.

The application of equation 24 to a specific motor is given later and is shown in Figure 5B by points A and B.

CIRCLE DIAGRAM FOR OPEN CIRCUIT CURRENT, OR K = INFINITY

At infinite operating speed or $K =$ infinity, the input current was found by equation 19c to be:

$$I_{1o} = \frac{D'(Z'_A + Z'_M)V}{Z'_A Z'_M} \tag{25}$$

If this current is subtracted from the total motor current of equation 16, the remaining current is

$$I_{eo} = \frac{V}{-\frac{Z_A Z'_A Z_M Z'_M}{(Z_A Z'_A + a^2 Z_M Z'_M)r_{2M}} + \frac{K^2(Z'_A Z'_M)^2}{(Z_A Z'_A + a^2 Z_M Z'_M)r_{2M}}} \tag{26}$$

Putting K = ± infinity in equation 26 will result in a value of zero for the current I_{eo}, thus suggesting an open circuited load. The combination of equations 25 and 26 represents a cantilever circuit as shown in Figure 5A(b).

The constants of this circuit are from equations 26 and 27,

$$\frac{C_o}{A_o} = \frac{D'(Z'_A + Z'_M)}{Z'_A Z'_M} \tag{27a}$$

$$A_o B_o = - \frac{Z_A Z'_A Z_M Z'_M}{(Z_A Z'_A + a^2 Z_M Z'_M) r_{2M}} \tag{27b}$$

$$A_o^2 Z_o = \frac{K^2 (Z'_A Z'_M)^2}{(Z_A Z'_A + a^2 Z_M Z'_M) r_{2M}} \tag{27c}$$

$$A_o = - \frac{Z'_A Z'_M}{Z_A Z'_A + a^2 Z_M Z'_M} \tag{27d}$$

$$Z_o = \frac{(Z_A Z'_A + a^2 Z_M Z'_M) K^2}{r_{2M}} \tag{27e}$$

$$B_o = \frac{Z_A Z_M}{r_{2M}} \tag{27f}$$

$$C_o = \frac{D(Z'_A + Z'_M)}{Z_A Z'_A + a^2 Z_M Z'_M} \tag{27g}$$

$$D_o \quad \frac{1 + B_o C_o}{A_o} = \frac{D(Z_A + Z_M)}{r_{2M}} \tag{27h}$$

Now let the equivalent load impedance $Z_o = R_o K^2 / \overline{\phi_o}$, where $R_o K^2$ = absolute value of Z_o. Then equations 25

and 26 can be combined and arranged in the form of the circle diagram equation (see equation 12) as follows:

$$I_1 = \frac{C_0 V}{A_0} + \frac{\frac{V}{A_0^2}\underline{/\phi_0}}{\frac{B_0}{A_0}\underline{/\phi_0} + R_0 K^2} \qquad (28)$$

As already outlined, the term $B_0/A_0\underline{/\phi_0}$ may be expanded into rectangular form $r_0 + jx_0$ from which the circle diameter becomes equal to

$$\frac{\frac{V}{A_0^2}\underline{/\phi_0}}{jx_0}$$

The application of equation 28 to a specific motor is outlined later and is shown by points C and D of figure 5B.

CIRCLE DIAGRAM FROM BLOCKED ROTOR CURRENTS, OR K = 0

Another form of an equation which gives a circle diagram is equation 29, in which the first term represents the input current for the blocked-rotor condition of the motor. The entire equation represents a cantilever circuit as shown in Figure 5A(c).

Equation 29 is of the form

$$I_1 = \frac{D_B V}{B_B} + \frac{V}{-A_B B_B - \frac{B_B^2}{Z_B}} \qquad (29)$$

or

$$I_1 = I_{1B} + I_{cB} \qquad (30a)$$

or

$$I_{eB} = I_1 - I_{1B} \tag{30b}$$

The block-rotor current is given by equation 17c. This current should be subtracted from the total current of equation 16 as indicated by equation 30b. Thus:

$$I_{eB} = \frac{V}{\dfrac{Z_A Z_A' Z_M Z_M'}{(Z_A Z_A' + a^2 Z_M Z_M')r_{2M}} - \dfrac{(Z_A Z_M)^2}{K^2(Z_A Z_A' + a^2 Z_M Z_M')r_{2M}}} \tag{31}$$

It will be noticed that equation (31) has all the properties of a circle diagram, as it is similar to the previous two circle diagram equations. Comparing this result with the general form of equation 29, it is apparent that

$$\frac{D_B}{B_B} = \frac{D(Z_A + Z_M)}{Z_A Z_M} \tag{32a}$$

$$A_B B_B = \frac{Z_A Z_A' Z_M Z_M'}{(Z_A Z_A' + a^2 Z_M Z_M')r_{2M}} \tag{32b}$$

$$\frac{B_B^2}{Z_B} = \frac{(Z_A Z_M)^2}{K^2(Z_A Z_A' + a^2 Z_M Z_M')r_{2M}} \tag{32c}$$

$$Y_B = \frac{1}{Z_B} = \frac{r_{2M}}{K^2(Z_A Z_A' + a^2 Z_M Z_M')} = \frac{1}{Z_o} \tag{32d}$$

$$B_B = \frac{Z_A Z_M}{r_{2M}} = B_o \tag{32e}$$

$$A_B = -\frac{Z_A' Z_M'}{Z_A Z_A' + a^2 Z_M Z_M'} = A_o \tag{32f}$$

$$D_B = \frac{D(Z_A + Z_M)}{r_{2M}} = D_0 \qquad (32g)$$

Now let equation 29 be changed to the form

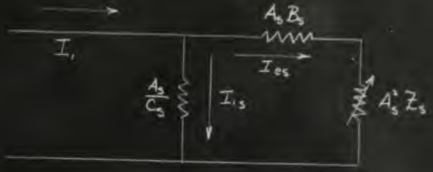
$$I_1 = \frac{D_B V}{B_B} + \frac{\dfrac{V}{B_B^2}}{-\dfrac{A_B}{B_B} - Y_B}$$

and finally, by putting

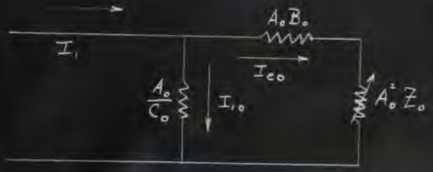
$$Y_B = \frac{G_B}{K^2}\underline{/\phi_B}$$

$$I_1 = \frac{D_B}{B_B}V + \frac{\dfrac{V}{B_B^2}\overline{/\phi_B}}{\dfrac{A_B}{B_B}\overline{/\phi_B} - \dfrac{G_B}{K^2}} \qquad (33)$$

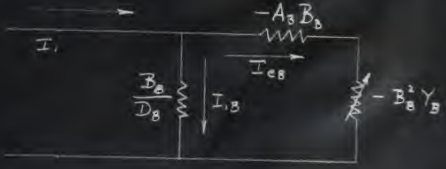
Computations also have been carried out for the circle diagram as outlined above, and it is seen that all three methods give the same final diagram. This last method is illustrated by points E and F of Figure 5B.



(a) Synchronous speed currents



(b) Open circuit currents



(c) Blocked rotor currents

Fig. 5A - Cantilever circuits representing circle diagram equations.

EXAMPLE SOLUTION OF CIRCLE DIAGRAM FOR A CAPACITOR MOTOR

The following data is taken from reference 7, page 617.

General Data: A four pole, one-fourth horsepower, 60 cycle, 110 volt, capacitor motor with core loss equal to 24 watts and friction and windage loss 13 watts is used.

Main winding

$Z_{1M} = 2.02 + j2.79 = 3.45 \underline{/54.1^\circ}$

$jX_{MM} = j66.8 = 66.8 \underline{/90^\circ}$ (core loss neglected)

Auxiliary winding

$Z_{1A} = 7.13 + j3.22 = 7.83 \underline{/24.35^\circ}$

$jX_{MA} = j92.9 = 92.9 \underline{/90^\circ}$ (core loss neglected)

$Z_c = 9 - j172$

$a = \frac{n_A}{n_M} = 1.18 \qquad a^2 = 1.39$

The core loss of 24 watts is taken into account by estimating the resistive element of the magnetizing impedances. It was shown by Puchstein and Lloyd that the main-winding magnetizing current is 1.51 amperes. If it is assumed that the core loss is divided equally between the two windings, the exciting resistance may be estimated as

$$R_{MM} = \frac{12}{(1.5)} = 5.3 \text{ ohms}$$

The corresponding resistance in the auxiliary winding is

$$R_{MA} = 1.39 \cdot 5.3 = 7.47$$

Finally,

$$Z_{MM} = 5.3 + j66.8 = 67.1\ \underline{/85^\circ}$$

$$Z_{MA} = 7.47 + j93 = 93.5\ \underline{/85^\circ}$$

Rotor circuit

$$Z_{2M} = 4.12 + j2.12 = 4.62\ \underline{/27.25^\circ}$$

$$Z_{2A} = 5.74 + j2.95 = 6.43\ \underline{/27.25^\circ}$$

Computations for the abbreviating terms are included at the end of this paper.

With the foregoing detail computations available the specific circle diagrams can be developed as follows: taking

$V = 110\ \underline{/90^\circ}$ as the reference vector.

SYNCHRONOUS SPEED. K = 1

$$I_{1s} = \frac{C_s V}{A_s} = \frac{0.000353\ /\ 1.5^\circ \cdot 110\underline{/90^\circ}}{0.0256\ \underline{/83.5^\circ}} = 1.52\ \underline{/8^\circ}$$

$$\frac{V\ \underline{/\phi_s}}{A_s^2} = \frac{110\ \underline{/90^\circ}\underline{/170.5^\circ}}{0.000655\underline{/167^\circ}} = 168{,}000\underline{/93.5^\circ}$$

$$\frac{B_s\ \underline{/\phi_s}}{A_s} = \frac{224\overline{/17.9^\circ}\underline{/170.5^\circ}}{0.0256\ \underline{/83.5^\circ}} = 8{,}750\ \underline{/69.1^\circ}$$

$$= 3{,}120 + j8{,}200$$

$$\text{Diameter of circle} = \frac{168{,}000\ \underline{/93.5^\circ}}{8{,}200\ \underline{/90^\circ}}$$

$$= 20.5\ \underline{/3.5^\circ}$$

From the value of I_{1s} and the diameter, it is possible

to locate points A and B on the circle diagram of Figure 5B.

OPEN CIRCUIT CURRENT. K = Infinity

$$I_{10} = \frac{C_o V}{A_o} = \frac{0.00589 \overline{/92.5^\circ} \cdot 110 /90^\circ}{0.0316 /27.4^\circ} = 20.5 \underline{/24.9^\circ}$$

$$= 18.6 + j8.65$$

$$\frac{V}{A_o^2} \, \underline{/\emptyset_o} = \frac{110 /90^\circ /170.5^\circ}{0.001 /54.8^\circ} = 110{,}000 \overline{/44.7^\circ}$$

$$\frac{B_o}{A_o} \, \underline{/\emptyset_o} = \frac{335 \overline{/48.5^\circ} /170.5^\circ}{0.0316 /27.4^\circ} = 10{,}600 \underline{/149.4^\circ}$$

$$= -9{,}150 + j5{,}360$$

$$\text{Diameter of circle} = \frac{110{,}000 \overline{/44.7^\circ}}{5{,}360 \underline{/90^\circ}}$$

$$= 20.5 \overline{/134.7^\circ}$$

The values of I_{10} and diameter previously given now are used to locate points C and D of Figure 5B. The diameter as computed is seen to check with previous calculations.

BLOCKED ROTOR CURRENT. K = 0

The equivalent circuit is that of Figure 5A(c). According to equations 32 the constants A_B, B_B, C_B, D_B, and Z_B are equal to the constants A_o, B_o, C_o, D_o, and Z_o of the case outlined under "Open Circuit Current."

Thus

$$I_{1B} = \frac{D_B V}{B_B} = \frac{42.2\ \overline{/87.1^\circ} \cdot 110/90^\circ}{335/48.5^\circ} = 13.9\ \underline{/51.4^\circ}$$

$$\frac{V\ \overline{/\phi_B}}{B_B^2} = \frac{110\ /90^\circ\ \overline{/170.5^\circ}}{112{,}000\ /97^\circ} = 0.000981\ \underline{/16.5^\circ}$$

$$-\frac{A_B}{B_B}\ \overline{/\phi_B} = -\frac{0.0316\ \overline{/27.4^\circ}\ \overline{/170.5^\circ}}{335\ /48.5^\circ}$$

$$= 0.0000944\ \underline{/30.6^\circ} = 0.0000811 + j0.000048$$

$$\text{Diameter of circle} = \frac{0.000981\ /16.5^\circ}{0.000048\ \underline{/90^\circ}}$$

$$= 20.5\ \overline{/73.5^\circ}$$

The values of I_{1B} and diameter just given now are used in locating points E and F of Figure 5B.

Points B, D, and F of Figure 5B correspond to the following values of K:

(a) Point B, see equation 24.

$$\frac{R_s}{1-K^2} + r_s = 0$$

or

$$K^2 = 1 + \frac{R_s}{r_s} = 1 + \frac{7{,}080}{3{,}120} = 3.37$$

$$K = 1.835$$

(b) Point D, see equation 28.

$$R_0 K^2 + r_s = 0$$

or

$$K^2 = -\frac{r_3}{R_0} = -\frac{-9,150}{7,080} = 1.29$$

$$K = 1.134$$

(c) Point F, see equation 33.

$$\frac{G_B}{K^2} + g_B = 0$$

or

$$K^2 = -\frac{G_B}{g_B} = -\frac{-0.000141}{0.0000811} = 1.74$$

$$K = 1.32$$

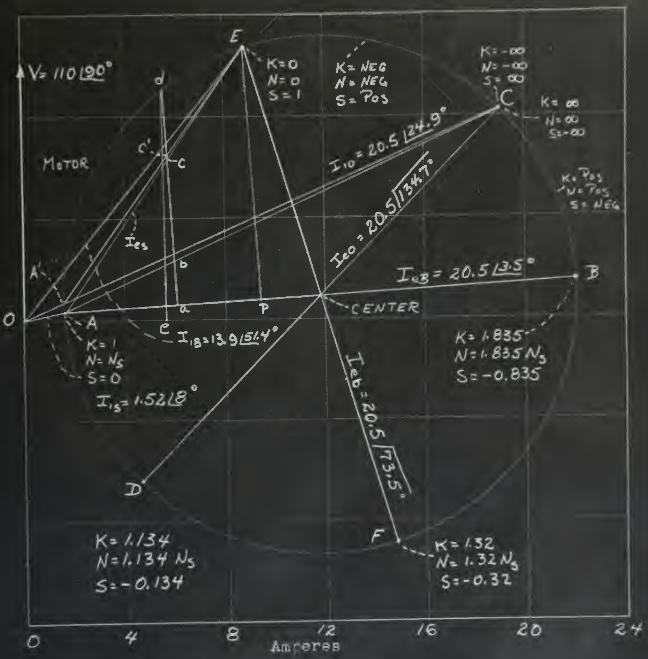


Fig. 5B - Circle diagram of capacitor motor.

GENERAL SUMMARY OF CIRCLE DIAGRAM

Point A corresponds to synchronous speed operation. The unit can turn only at synchronous speed if driven mechanically. However, the stator magnetic field still must be supplied from the stator input current I_{1s}.

Proceeding around the circle in a clockwise manner, the motor speed is seen to decrease until blocked rotor is reached as indicated by point E, where K = 0, or slip = 1. Thus the motor region of the circle is limited to the arc AE.

From point E to point C, the rotor is indicated as operating in a direction reverse to that for straight motor action (arc AE). This action can take place only by the driving force of an external prime mover or a mechanical load which has become a prime mover. Since the stator still is drawing power from the a-c supply circuit, it appears that the motor now is acting as an energy-absorption device. At point C the speed is infinity in the reverse direction to normal motor action.

Returning to point A and proceeding counterclockwise around the circle, we notice that K becomes steadily greater than unity, indicating operation above synchronous speed, the rotor turning in the same direction as that for regular motor action. Infinite speed again is obtained at point C, approached in a counterclockwise fashion. This lower region of the circle corresponds to generator

action.

To obtain the operating characteristics from Figure 5B, it is first necessary to draw a sufficiently large number of lines from the circle and perpendicular to the diameter AB, such as ad or EP. The motor section of Figure 5B is shown in more detail in Figure 5D. The circle envelope of the vector I_{es} applies to a simple equivalent series circuit with applied voltage equal to $V_e = (V \div A_s^2)\underline{/\phi_s}$ and impedance $(B_s \div A_s)\underline{/\phi_s} + (R_s \div [1-K^2])$ (see equation 24) and illustrated by the circuit sketch of Figure 5D. From equation 24 and the data previously worked out

$$I_{es} = \frac{168/93.5^\circ}{3.12 + \frac{7.08}{1-K^2} + j8.2}$$

Power is absorbed by the two resistances of this circuit, namely, $R_1 = 3.12$ ohms, and $R_{2e} = 7.08 \div (1-K^2)$ ohms, the total power being proportional to the inphase component of current as illustrated by the lines (ad).

For zero speed or K = 0, the two resistances are $R_1 = 3.12$ ohms and $R_2 = 7.08$ ohms or a total of 10.2 ohms. Thus at K = 0, the total absorbed power is proportional to the length of PE 10.05 amperes.

The point M can be obtained in two ways. From previous analysis it was shown that line AC represents the value of I_{es} for K = infinity, under which condition the resistance $R_{2e} = 0$. The intersection of line AC and line PE (Figures 5B and 5C) thus determines the point M. Point

M also may be obtained by making the ratio $\frac{EM}{MP}$ equal to the ratio 7.08/3.12. Thus it follows that

$EM = 6.95$ amperes, while $MP = 3.07$ amperes.

For any operating point d, the circuit resistance can be expressed as $R_0 + R_1 + R_2$, where $R_1 + R_2 = 3.12 + 7.08 = 10.2$ ohms from which $R_0 = 7.08\ K \div (1-K^2)$ or $K^2 = R_0 \div (R_0 + 7.08)$. The power absorbed by these three resistances (R_0, R_1, and R_2) will be proportional to the lengths cd, ab, and bc of Figure 5D, from which it follows that:

$$K^2 = \frac{cd}{bd} \quad \text{or } K = \sqrt{\frac{cd}{bd}}$$

From Figure 5D it is possible to obtain certain operating characteristics as follows:

Gross output = V (cd) watts

Equivalent rotor copper loss = V (bc) watts

Rotor input = V (bd) watts

Stator input = V (de) watts

(de is taken parallel to vector V)

Ratio of operating to synchronous speed

$$K = \sqrt{\frac{cd}{bd}}$$

Input current = Od Amperes

Power Factor in per unit = $\frac{de}{Od}$

The net power output and net efficiency may be approximated by introducing line $A'E$ Figure 5B where A' repre-

sents the no load operating current on the circle. The line A E may be drawn as a straight line without introducing appreciable error.

Then from Figure 5B:

Net power output $=$ V $(c'd)$ watts

Net efficiency in per unit $= \frac{c'd}{de}$

The Resultant Motor characteristics are shown in Figure 5C. All were determined from the circle diagram of Figure 5D with the exception of torque, stator copper loss, and core loss.

Conclusions

The mathematical material of this paper may appear to be somewhat formidable. However, the circle diagram (Figures 5B and 5D) can be obtained from a combination of test and computed data as follows:

(a) Point A corresponds practically to no load operation.

(b) Point E can be obtained from a blocked rotor test.

(c) Point C must be computed according to equations 19. For general work it is possible to take $D' = 1$ with the simplification of $Z'_M = Z_{1M} + jx_{2M}$ and $Z'_A = Z_{1T} + jx_{2A}$.

(d) Points A, E, and C are sufficient to determine the circle. Other details then are carried out as indicated in Figure 5D.

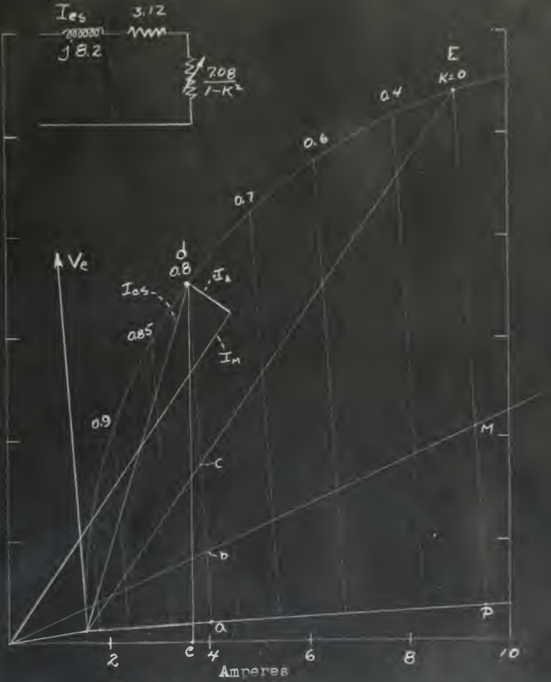


Fig 5D - Motor circle diagram.

A few identities and certain other simplifications which are used in the chapter on the circle diagram are tabulated as follows:

$$Z_{SM} = Z_{MM} + Z_{2M}, \quad Z_{PM} = Z_{MM} + Z_{1M}$$

$$Z_{SA} = Z_{MA} + Z_{2A}, \quad Z_{PA} = Z_{MA} + Z_{1T}$$

$$Z'_{SM} = Z_{MM} + jx_{2M}, = Z_{SM} - r_{2M}$$

$$Z'_{SA} = Z_{MA} + jx_{2A} = Z_{SA} - r_{2A}$$

$$Z_{SA} = a^2 Z_{SM}, \; Z'_{SA} = a^2 Z'_{SM}, \; Z_{MA} = a^2 Z_{MM}$$

$$D = \frac{Z_{SM}}{Z_{MM}} = \frac{Z_{SA}}{Z_{MA}}, \quad D' = \frac{Z'_{SM}}{Z_{MM}} = \frac{Z'_{SA}}{Z_{MA}}$$

$$D' = D - \frac{r_{2M}}{Z_{MM}} = D - \frac{r_{2A}}{Z_{MA}}$$

$$Z_M = \frac{Z_{SM} Z_{1M}}{Z_{MM}} + Z_{2M} = DZ_{1M} + Z_{2M}$$

$$Z_A = \frac{Z_{SA} Z_{1T}}{Z_{MA}} + Z_{2A} = DZ_{1T} + Z_{2A}$$

$$Z'_M = \frac{Z'_{SM} Z_{1M}}{Z_{MM}} + jx_{2M} = D' Z_{1M} + jx_{2M} = Z_M - A_M r_{2M}$$

$$Z'_A = \frac{Z'_{SA} Z_{1T}}{Z_{MA}} + jx_{2A} = D' Z_{1T} + jx_{2A} = Z_A - A_A r_{2A}$$

$$D' Z_M - DZ'_M = r_{2M}, \quad D' Z_A - DZ'_A = r_{2A} = a^2 r_{2M}$$

$$A_M = \frac{Z_{PM}}{Z_{MM}}, \quad A_A = \frac{Z_{PA}}{Z_{MA}}$$

The solution follows the definitions as outlined above, the results of which are substituted in equations 24, 28, and 33.

$$Z_{SM} = Z_{MM} + Z_{2M} = 5.3 + j66.8 + 4.12 + j2.12$$
$$= 9.42 + j68.92 = 69.7\angle 82^\circ$$

$$Z'_{SM} = Z_{MM} + jx_{2M} = 5.3 + j66.8 + j2.12 = 5.3 + j68.92$$
$$= 69.0\angle 85.5^\circ$$

$$Z_{SA} = Z_{MA} + Z_{2A} = 7.47 + j93 + 5.74 + j2.95$$
$$= 13.21 + j95.95 = 97\angle 82^\circ$$

$$Z'_{SA} = Z_{MA} + jx_{2A} = 7.47 + j93 + j2.95$$
$$= 7.47 + j95.95 = 96.0\angle 85.5^\circ$$

$$D = \frac{Z_{SM}}{Z_{MM}} = \frac{Z_{SA}}{Z_{MA}} = \frac{69.7\angle 82^\circ}{67.1\angle 85^\circ} = \frac{97\angle 82^\circ}{93.5\angle 85^\circ} = 1.04\angle\overline{3^\circ}$$

$$D' = \frac{Z'_{SM}}{Z_{MM}} = \frac{Z'_{SA}}{Z_{MA}} = \frac{69.0\angle 85.5^\circ}{67.1\angle 85^\circ} = \frac{96.0\angle 85.5^\circ}{93.5\angle 85^\circ} = 1.03\angle 0.5^\circ$$

$$Z_M = DZ_{1M} + Z_{2M} = 1.04\angle\overline{3^\circ} \cdot 3.45\angle\overline{54.1^\circ} + 4.12 + j21.12$$

$$Z_M = 6.38 + j4.88 = 8.04\angle 37.5^\circ$$

$$Z'_M = D' Z_{1M} + jx_{2M} = 1.03\angle 0.5^\circ \cdot 3.45\angle 54.1^\circ + j2.12$$

$$Z'_M = 2.08 + j4.99 = 5.4\angle 66.5^\circ$$

$$Z_A = DZ_{1T} + Z_{2A} = 1.04\angle\overline{3^\circ} \cdot 169\angle\overline{84.5^\circ} + 5.74 + j2.95$$

$$Z_A = 11.83 - j172.05 = 172.1\angle\overline{86.1^\circ}$$

$$Z'_A = 16.7 - j170.75 = 171\angle\overline{84.4^\circ}$$

Additional terms required are as follows:

$$Z_A Z'_A = 172.1/\overline{86.1^\circ} \cdot 171/\overline{84.4^\circ} = 29{,}400/\overline{170.5^\circ}$$
$$= -29{,}000 - j4{,}850$$

$$a^2 Z_M Z'_M = 1.39 \cdot 8.04/\underline{37.5^\circ} \cdot 5.4/\underline{66.5^\circ} = 60.3/\underline{104^\circ}$$
$$= -14.45 + j58.5$$

$$Z_A Z'_A + a^2 Z_M Z'_M = -29{,}014 - j4{,}792 = 29{,}200/\overline{170.5^\circ}$$

$$Z_A Z_M = 172.1/\overline{86.1^\circ} \cdot 8.04/\underline{37.5^\circ}$$
$$= 1{,}380/\overline{48.5^\circ} = 915 - j1{,}032$$

$$Z'_A Z'_M = 171/\overline{84.4^\circ} \cdot 5.4/\underline{66.5^\circ} = 923/\overline{17.9^\circ} = 876 - j285$$

$$Z_A + Z_M = 11.83 - j172.05 + 6.38 + j4.88$$
$$= 18.21 - j167.17 = 168/\overline{83.7^\circ}$$

$$Z'_A + Z'_M = 16.7 - j170.75 + 2.08 + j4.99$$
$$= 18.78 - j165.76 = 167/\overline{83.5^\circ}$$

$$D(Z_A + Z_M) = 1.04/\overline{3^\circ} \cdot 168/\overline{83.7^\circ} = 174/\overline{86.7^\circ}$$
$$= 8.96 - j173.5$$

$$D'(Z'_A + Z'_M) = 1.03/\underline{0.5^\circ} \cdot 167/\overline{83.5^\circ} = 172/\overline{83^\circ}$$
$$= 19.2 - j171.8$$

$$D(Z_A+Z_M) - D'(Z'_A + Z'_M) = -10.24 - j1.7 = 10.3/\overline{169^\circ}$$

$$Z_A Z_M - Z'_A Z'_M = 915 - j1{,}032 - 876 + j285 = 39 - j747 = 747/\overline{87^\circ}$$

CHAPTER VI

THE EQUIVALENT CIRCUIT(3)

The equivalent circuit of Figure 6A and the corresponding torque equation 1 are derived from the fundamental equations of the cross-field theory. It is as exact and general as the cross-field theory itself. The theory does not take into account the existance of two separate skin effects in rotor conductors or the effect of space harmonics. When these effects are considerable, the cross-field theory and, therefore, the equivalent circuit would not hold good. In general, if there are two or more coils per phase per pole, and if the frequency is not high enough to give unusual skin effects, the equivalent circuit can be expected to give accurate results.

Figure 6A is the equivalent circuit of a capacitor motor with both the capacitor phase and main phase connected to the same power source. However, in setting up an equivalent circuit in which current from both phases must feed into the rotor circuit, represented by one set of constants, such rotor currents must be represented as in phase with each other on the circuit to correspond to the actual motor condition of being in time quadrature. This can be done by using the j-operator on the capacitor winding applied voltage, even though it must be kept in mind that the two windings actually are connected to the same source. Because motor performance can be identified

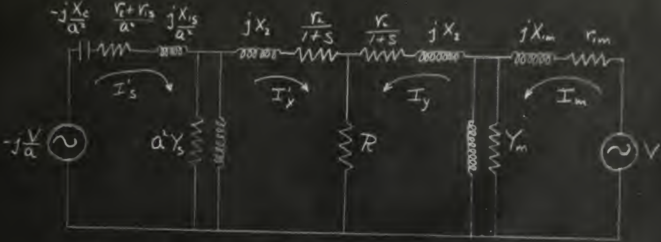


Fig. 6A - The equivalent circuit for the capacitor motor.

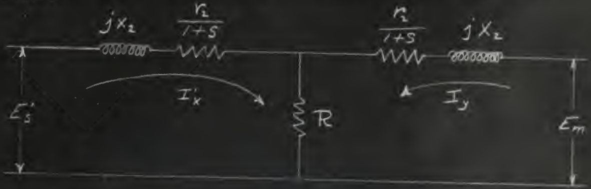


Fig. 6B - Equivalent rotor circuit of the capacitor motor.

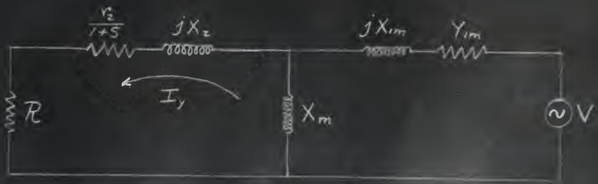


Fig. 6C - Approximate equivalent circuit at maximum torque point. The capacitor phase is assumed to supply its own magnetization current, the iron loss, and the friction and windage loss.

by volts per turn, the correct effect of the capacitor winding (with different number of turns) requires that the capacitor winding be divided by the ratio 'a'. Hence the actual applied voltage on the capacitor winding is multiplied by -j/a because of these considerations.

For any capacitor motor it is only necessary to solve the circuit of Figure 6A for I'_x, I_y, I'_s, I_m from which the torque, line current, efficiency, power factor, and so forth are calculated readily.

The iron loss is represented by the real components of Y_m and Y_s . Because the iron losses are proportional to the square of the air-gap voltages E_m and E_s for a good range of voltage, this can be expected to hold for the entire performance range, that is, $0.7 < K < 1$.

The friction and windage losses should be deducted from the generated torque.

$$T = RI^2 + \frac{2r_2}{1+K} I'_x I_y \cos(I'_x, I_y) \text{ synchronous watts} \quad (1)$$

The torque equation (1) consists of two terms. The first term is the power absorbed by the virtual resistance R. It represents the major part of running torque but vanishes as R vanishes at zero speed. The second term represents exclusively the starting torque. Its effect on the running torque usually amounts to only a few percent for an adequately designed motor. An examination of this second term shows:

1.) When the angle between I_x' and I_y is small, the second term is positive, and the generated torque in synchronous watts is higher than the energy absorbed by the virtual resistance R. Better efficiency would be realized under this condition.

2.) When the angle between I_x' and I_y is larger than 90 degrees, the second term is negative. The total current I is reduced also. The torque would be small, and the efficiency would be poor.

Physically, a small angle between I_x' and I_y means that the capacitor phase and the main phase are matched so that they supply the virtual load R jointly. If a flow of energy exists either from the capacitor phase to the main phase or vice versa, the angle between I_x' and I_y would be larger than 90 degrees, and the efficiency would be poor.

A relationship between the motor constants which guarantees satisfactory operation in this light will emerge from later developments.

The circuit of Figure 6A can be solved readily by taking the voltage and currents at the virtual resistor R as a starting point. Either looking into the right or left from this virtual resistor, we see a T-circuit connected to a power source. Let the voltage across R be called E. The voltage E_m across the magnetizing branch Y_m is then

$$E_m = E + I_y Z_2' \qquad (2)$$

The current in the magnetizing branch is $I_m - I_y$ and this current should be equal to E_mY_m. Therefore,

$$I_m = I_y + E_mY_m \tag{3}$$

The terminal voltage V is equal to E_m plus the voltage drop across the impedance Z_{im}.

$$I_mZ_{im} + E_m = V \tag{4}$$

substituting equation 2 in equation 3

$$I_m = I_y(1 + Y_mZ_2') + EY_m \tag{5}$$

substituting equation 5 in equation 4

$$V = E(1 + Z_{im}Y_m) + I_y(Z_2' + Z_{im} + Z_{im}Y_mZ_2') \tag{6}$$

Eliminating I_y between equation 5 and equation 6

$$V(1 + Y_mZ_2') - I_m(Z_2' + Z_{im} + Z_{im}Y_mZ_2') = E \tag{7}$$

Equations 6 and 7 can be written

$$\begin{aligned} I_y &= Y_{tm}V - Y_{om}E \\ &= I_{om} - Y_{om}E \end{aligned} \tag{8}$$

$$\begin{aligned} I_m &= Y_{em}V - Y_{tm}E \\ &= I_{em} - Y_{tm}E \end{aligned} \tag{9}$$

where

$$Y_{tm} = \frac{1}{Z_2' + Z_{im} + Z_{im}Z_2'Y_m} \tag{10}$$

$$Y_{om} = (1 + Z_{im}Y_m)Y_{tm} \tag{11}$$

$$Y_{em} = (1 + Z_2'Y_m)Y_{tm} \tag{12}$$

$$I_{om} = Y_{tm}V \tag{13}$$

$$I_{em} = Y_{em}V \tag{14}$$

The same procedure can be applied to the circuit to the left of R and results in

$$I_x' = I_{os} - Y_{os}E \qquad (15)$$

$$I_s' = I_{es} - Y_{ts}E \qquad (16)$$

where

$$Y_{ts} = \frac{1}{Z_2' + Z_{is}' + Z_{is}' Z_2' a^2 Y_s} \qquad (17)$$

$$Y_{os} = (1 + Z_{is}' a^2 Y_s) Y_{ts} \qquad (18)$$

$$Y_{es} = (1 + Z_2' a^2 Y_s) Y_{ts} \qquad (19)$$

$$I_{os} = Y_{ts}(-jV/a) \qquad (20)$$

$$I_{es} = Y_{es}(-jV/a) \qquad (21)$$

adding equations 15 and 16

$$\begin{aligned} I &= I_x' + I_y \\ &= (I_s + I_{om}) - E(Y_{os} + Y_{om}) \qquad (22) \\ &= I_o - EY_o \end{aligned}$$

where

$$I_o = I_s + I_{om} \qquad (23)$$

$$Y_o = Y_{os} + Y_{om} \qquad (23a)$$

From equations 9 and 16

$$I_{line} = I_m + \frac{jI_s'}{a} = (I_{em} + \frac{jI_{es}}{a}) - (Y_{tm} + \frac{jY_{ts}}{a})E \qquad (24)$$

substituting the relation $E = I R$ in equation 22 results in

$$I = \frac{I_o}{1 + RY_o} \qquad (25)$$

For a given motor at a given speed I_o and Y_o can be calculated by going through equations 10-14, 17-21 and equations 23 and 23a. From equation 25 can get I and E. From equations 8, 9, 15, and 16, I_y, I_m, I_x', I_s' can be calculated. The generated torque is calculated from

equation 1. The efficiency and power factor are evaluated from I_m, I_s and the torque, but Z_2' is dependent on K through the term $r_2/(1+K)$. That means that all the Y's are dependent on K, and it is therefore necessary to recalculate for every point on the performance curve if an exact result is required. However, sufficiently accurate results can be obtained by using

$$Z_2' = 0.55r_2 + jx_2 \qquad (26)$$

As K varies from 0.7 to 1, $r_2/(1+K)$ varies from $0.59r_2$ to $0.50r_2$. The approximate equation 26 introduces an error of at most $0.05r_2$ to Z_2'. Its effect on the currents and torque are well below one percent.

With the introduction of the approximate equation 26, it is only necessary to calculate equations 10-14 and 17-21 once for a motor. After that, calculation of performance is no more complicated than for a balanced polyphase motor.

The torque expression corresponding to equation 26 is

$$T = RI^2 + 1.1r_2 I_x' I_y \cos(I_x', I_y) \qquad (27)$$

THE MAXIMUM TORQUE

Using the circuit of Figure 6A and the torque expression of equation 1 in trying to derive an exact expression for maximum torque leads to complicated calculations with third order equations. But such a procedure is hardly justifiable because some unaccountable factors may

introduce an error far larger than an approximation of Figure 6A and equation 1 introduce. These factors are:

1. The two separate skin effects of the rotor.
2. The variation of stator and rotor resistances as a result of temperature variations.
3. The effect of space harmonics.
4. The variations of stator and rotor. This variation is especially appreciable under successive starting.

The combined effects of these factors introduce an error which may well amount to a few percent.

In general, at speed of maximum torque, the major portion of rotor current is supplied from the main winding. It can be assumed without much error that

1. The capacitor phase supplies the cross-field magnetizing current and the portion of energy that overcomes friction and windage losses as well as iron losses, while the main phase alone supplies the power that converts into a measureable torque. With this assumption, the equivalent circuit becomes that of Figure 6C.
2. The maximum torque occurs at the speed of maximum rotor input.

From Figure 6C it is clear that not all the rotor input converts into torque expressed in synchronous watts.

$$\text{Rotor input} = I_y^2 \left(R + \frac{r_2}{1 + K}\right) \qquad (28)$$

$$\text{Synchronous watts} = I_y^2 R \qquad (29)$$

Dividing equation 29 by equation 28

$$\frac{\text{Synchronous watts}}{\text{Rotor input}} = \frac{R}{R + \frac{r_2}{1+K}} = \frac{\frac{Kr_2}{1-K^2}}{\frac{Kr_2}{1-K^2} + \frac{r_2}{1+K}} = K \qquad (30)$$

Equation 30 is useful in establishing a relation between the maximum torque of a capacitor motor and that of its corresponding polyphase motor. The methods of calculating the latter is well developed. Let T_{2max} be the maximum torque of a straight 2 phase motor with each of its windings exactly the same as the winding of the capacitor motor. Then T_{2max} is equal to the maximum rotor input of this 2 phase motor or two times the maximum rotor input of the capacitor motor. From equation 30

$$T_{max} = K \frac{T_{2max}}{2} \qquad (31)$$

Equation 31 has been verified for a number of cases. The agreement between the measured and calculated values turned out to be always within ten percent.

The starting torque of a capacitor motor is obtained readily by substituting $K = 0$ into equation 1.

Starting torque $= 2r_2 I'_{x0} I_{y0} \cos(I'_{x0}, I_{y0})$ where I'_{x0}, I_{y0} are values of I'_x, I_y at $K = 0$.

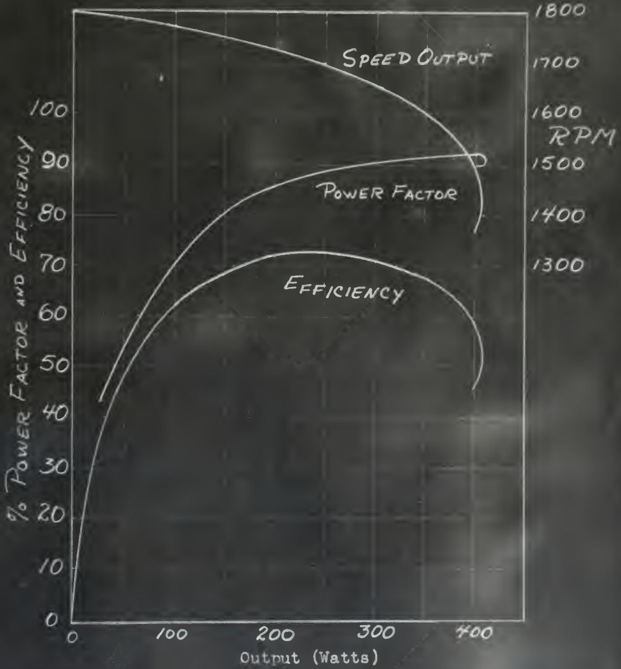


Fig. 6D - Performance curves of Mr. Morrill's ¼ horsepower, 4 pole motor. Using the equivalent circuit.

NOMENCLATURE FOR EQUIVALENT CIRCUIT

V = applied volts (Used as reference vector)

E_m = counter emf in the main axis

$Z_{im} = r_{im} + jX_{im}$
= local impedance of the main stator winding

$Z_2 = r_2 + jX_2$
= rotor impedance in main winding terms

E_s = counter emf of the starting winding

$Z_{is} = r_{is} + jX_{is}$
= local impedance of the stator starting winding

$Z_c = r_c - jX_c$
= capacitor impedance

I_m = stator current, main axis

I_s = stator current, starting axis

I_y = stator component of the rotor current, main axis

I_x = stator component of the rotor current, starting axis

Z_s = magnetizing impedance in the S-axis

Z_m = magnetizing impedance in the M-axis

Y_m = magnetizing admittance of main winding $= 1/Z_m$

Y_s = magnetizing admittance of capacitor winding $= 1/Z_s$

$$a = \frac{\text{effective starting winding turns}}{\text{effective main winding turns}}$$

$$K = \frac{\text{actual revolutions per minute}}{\text{synchronous revolutions per minute}}$$

$Z'_2 = r_2/(1+K) + jX_2$
= (Its approximate expression $0.55r_2 + jX_2$)

R = virtual load resistance

$$= \frac{Kr_2}{1 - K^2}$$

I_{om} = short-circuit current at R from main phase alone

I_{em} = values of I_m when R is short-circuited

I_{os} = short-circuit current at R from capacitor phase alone

I_{es} = values of I_s' when R is short-circuited

I_o = short-circuit current at $R = I_{os} + I_{om}$

I = current through virtual load R

E = back electromotive force across R

E_{om} = main phase open-circuit voltage, defined as I_{om}/Y_{om}

E_{os} = capacitor phase open-circuit voltage, defined as I_{os}/Y_{os}

I_s' = capacitor winding current in main winding terms with 90 degrees backward phase shift $= -jaI_s$

I_x' = capacitor axis rotor current in main terms with 90 degrees backward phase shift $= -jaI_x$

T = d-c component of the generated torque

T_{max} = maximum d-c torque of the capacitor motor

T_{2max} = maximum torque of the 2-phase motor, the windings of which are identical with the main winding of the capacitor motor

F & W = friction and windage

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